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ANN-Based Global 21-cm Signal Extraction with Chromatic Ionospheric Effects

Anshuman Tripathi

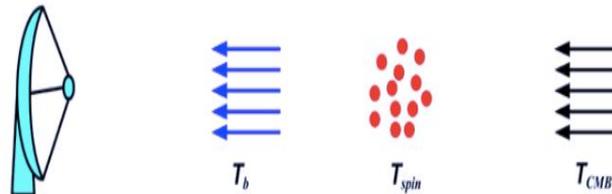
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Observation of 21-cm Signal

$$\delta T_b(\nu) = \frac{T_s - T_\gamma}{1 + z} (1 - \exp^{-\tau_{\nu 0}})$$

$$\delta T_b \approx 27(1 - x_i) \left(\frac{\Omega_{b,0} h^2}{0.023} \right) \left(\frac{0.15}{\Omega_{m,0} h^2} \frac{1+z}{10} \right)^{1/2} \left(1 - \frac{T_\gamma}{T_s} \right)$$

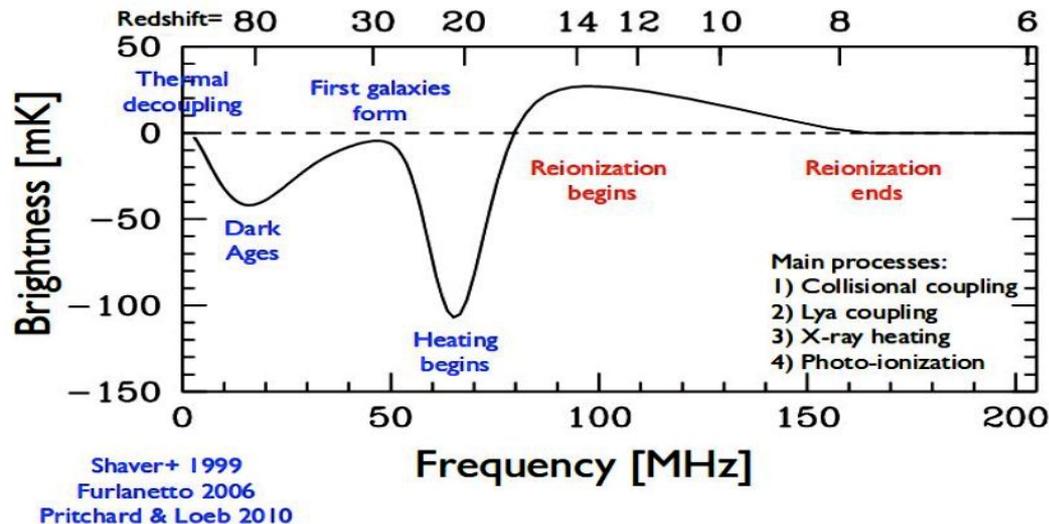


Field 1959; Pritchard & Loeb 2012

Observing type: Single radiometer

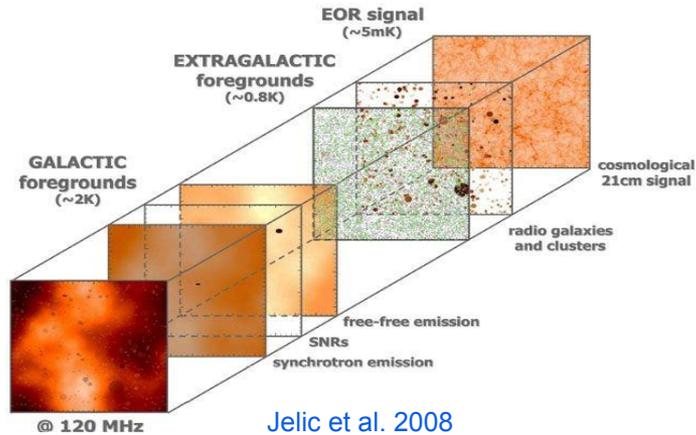
Signal: Global 21-cm Signal

Experiments: EDGES, SARAS, BigHorns, LEDA, REACH, MIST, SEAMS, PRATUSH



21CM Cosmology: Observational Challenges

- Galactic and Extragalactic Foreground



- Earth's Ionosphere

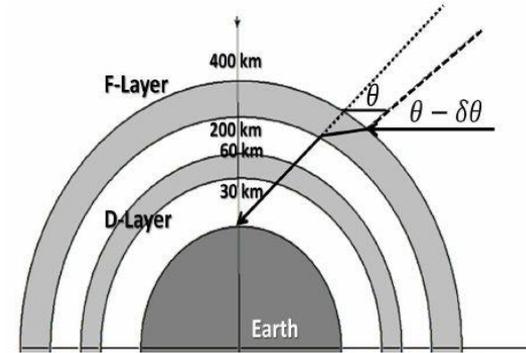


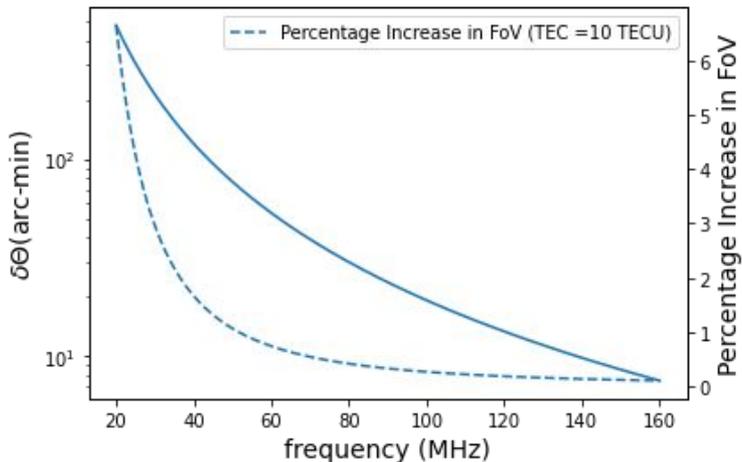
Fig : Schematic representation of the ionosphere showing the F- and D- layers (not scaled), which show the refraction. (Datta et al. 2016)

- Radio Frequency Interference (RFI)
- Other Systematics effects

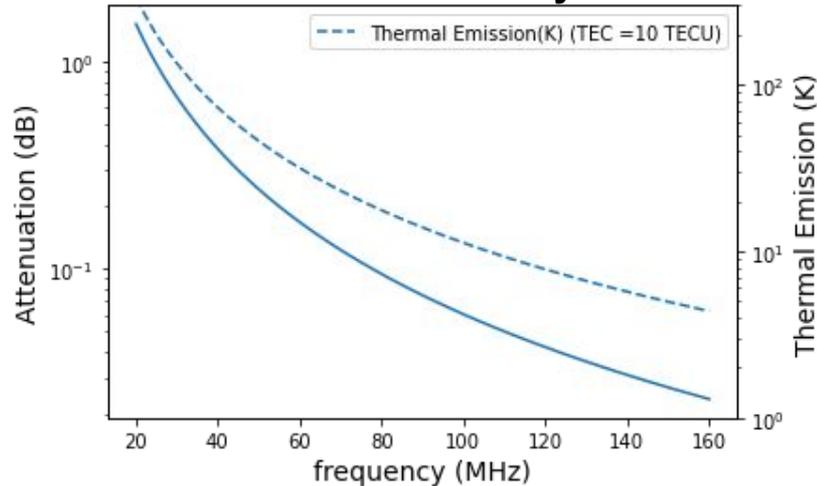
- In the lower frequency the ionosphere affect the antenna beam which lead to chromatic mixing of spatial structure into spectral structure. ([Vedantham et. al 2014](#), [Datta et al. 2016](#)).
- It can couple the relatively large angular fluctuations in the galactic foreground into spectral structures that may confuse global 21-cm signatures. ([Mozdzen et al. \(2016\)](#))
- It effects are significant especially with the Galactic Centre overhead.
- In [E. Shen et al. 2022](#), it is demonstrated that, under turbulent ionospheric conditions, an error exceeding 5 percent in our understanding of the ionospheric parameters could result in false or null detection.

21CM GLOBAL SIGNAL: Effect of Ionosphere

Ionospheric Refraction due to F-layer



Ionospheric Absorption and Thermal Emission due to D-layer



Increase in the effective FoV

$$T_{\text{sky}}^{\text{iono}}(\nu, t; \Theta_0, \Phi_0) = \int_0^{2\pi} d\Phi$$

$$\times \int_0^{\pi/2} d\Theta B'(\nu, \Theta - \Theta_0 - \delta\theta(t), \Phi)$$

$$\times T_{\text{sky}}(\nu, \Theta - \Theta_0, \Phi - \Phi_0) \sin \Theta$$

Signal (T_{21}) + Foreground (T_{FG})

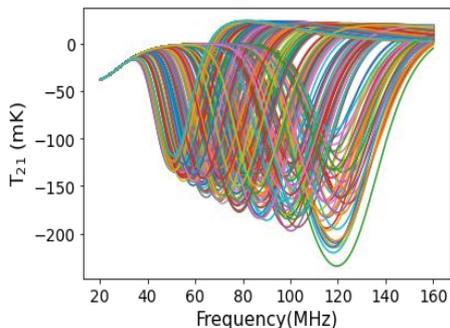
$$T_{\text{Ant}}^{\text{iono}}(\nu, \text{TEC}(t), \Theta_0, \Phi_0) = T_{\text{sky}}^{\text{iono}}(\nu, t; \Theta_0, \Phi_0)$$

$$\times (1 - \tau(\nu, \text{TEC}(t)))$$

$$+ \tau(\nu, \text{TEC}(t)) * \langle T_e \rangle$$

(Tripathi A. et al. 2024a, MNRAS)

Training Data Sets



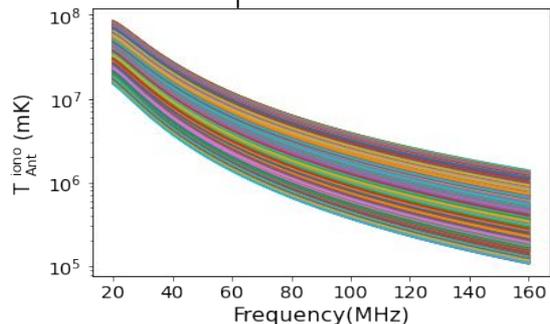
We use the Accelerated Reionization Era Simulations (ARES) code was designed to rapidly generate models for the global 21- cm signal.

(Mirocha et al, 2012, 2015).

3rd order $\log(T) - \log(\nu_0)$ polynomial model

+

Foreground
+
Ionospheric
effects



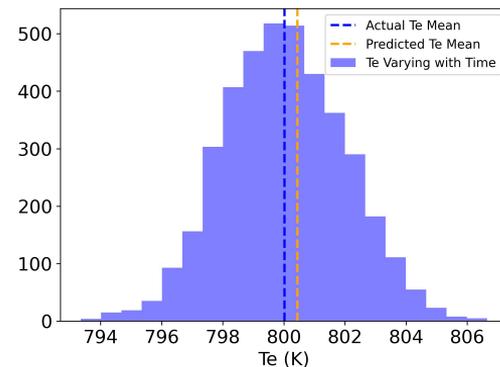
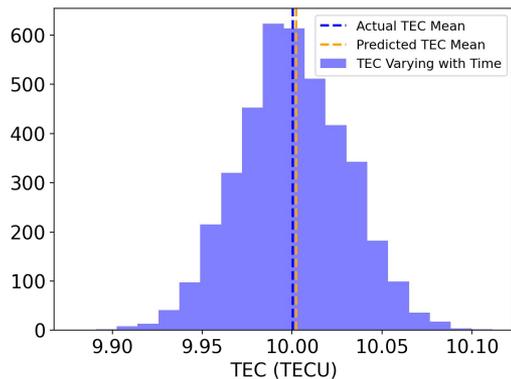
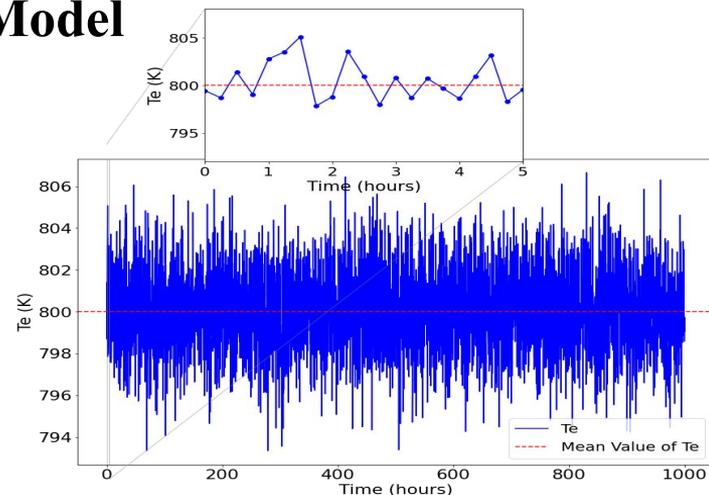
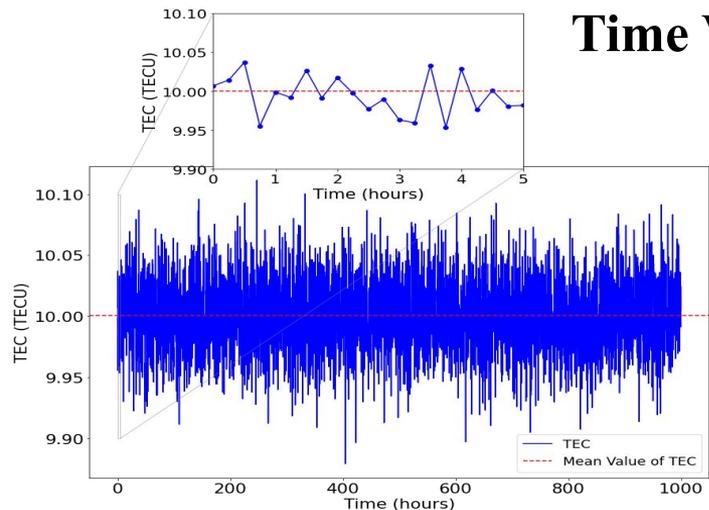
| Parameters | R2 Score | RMSE |
|------------|----------|--------|
| J_{ref} | 0.9614 | 0.0628 |
| J_{z0} | 0.9738 | 0.0605 |
| X_{z0} | 0.9713 | 0.0616 |
| T_{z0} | 0.9668 | 0.0577 |
| J_{dz} | 0.9634 | 0.0608 |
| X_{dz} | 0.9578 | 0.0658 |
| T_{dz} | 0.9595 | 0.0636 |
| a_0 | 0.9810 | 0.0439 |
| a_1 | 0.9655 | 0.0590 |
| a_2 | 0.9586 | 0.0642 |
| a_3 | 0.9610 | 0.0625 |
| TEC | 0.9658 | 0.0578 |
| T_e | 0.9728 | 0.0588 |

ANN Model

Saved ANN
Model

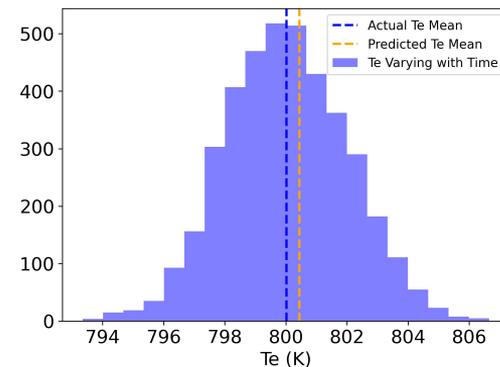
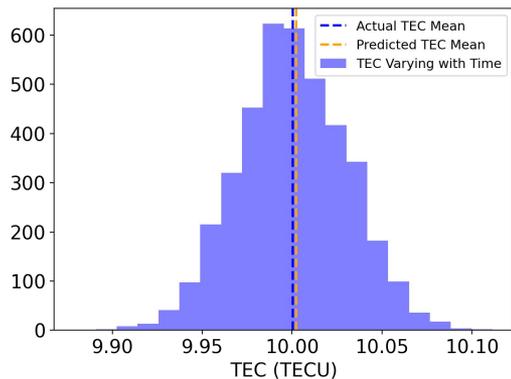
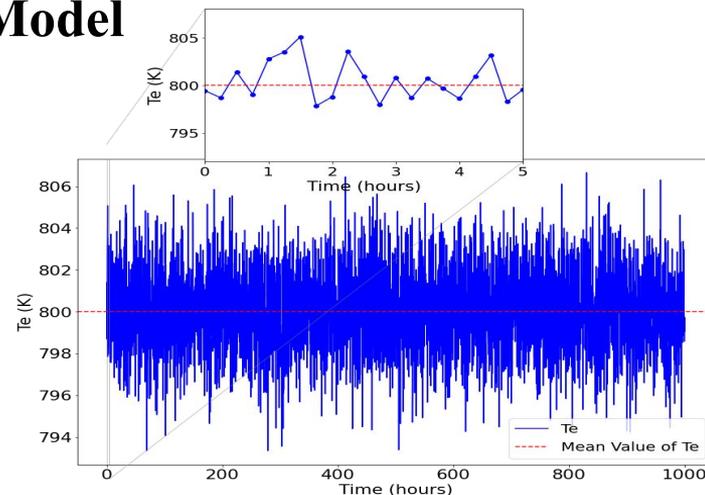
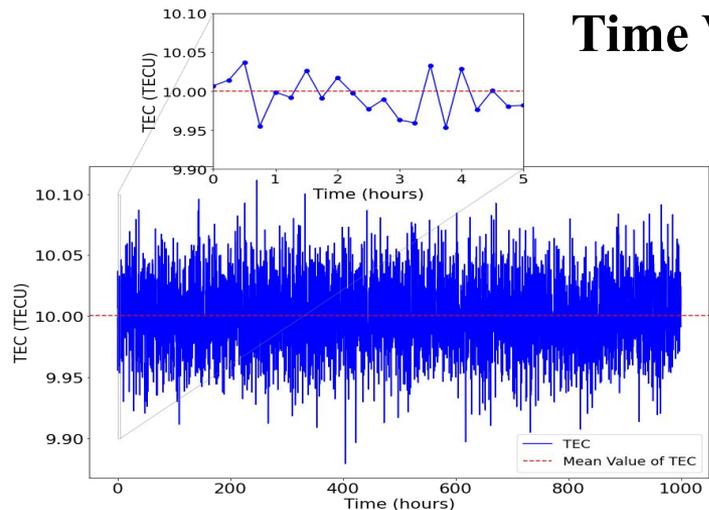
Test Data set
(Signal + Foreground
+ Ionospheric effect +
Thermal Noise)

Time Varying Ionosphere Model



Tripathi A. et al. 2024a, MNRAS

Time Varying Ionosphere Model



Tripathi A. et al. 2024a, MNRAS

Conclusion: Simple ANN is robust in 21cm signal parameter estimates in presence of slowly varying ionosphere.

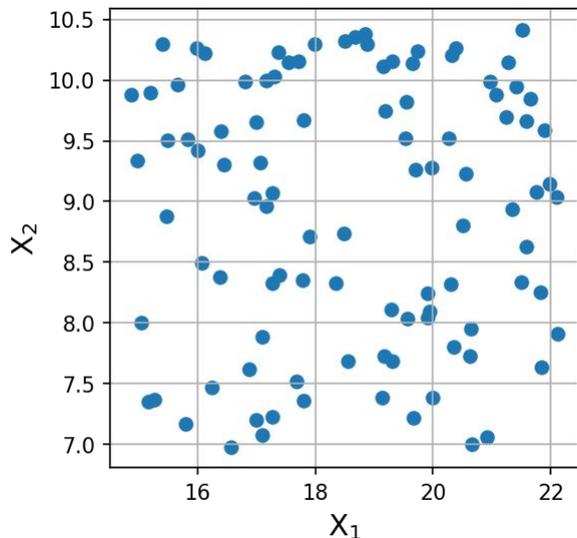
Motivation

- Without any observational constraints, the signal parameter space is enormously huge.
- Non-parametric techniques may produce biased inferences as they explore only a limited parameter space. To avoid bias, the data set includes multiple realizations of the signal generated by covering the whole parameter space.
- To reduce the computational burden of training Artificial Neural Networks (ANNs) and to achieve greater accuracy in extracting parameters from signals, it is necessary to employ sampling methods.

Charting Parameters Space using Sampling Technique

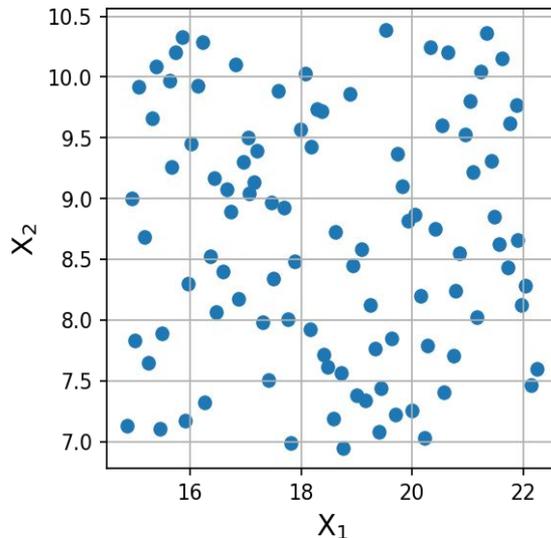
Random Sampling (Rand)

Samples assuming a uniform distribution. It may not be suitable for exploring high-dimensional spaces or regions of low probability.



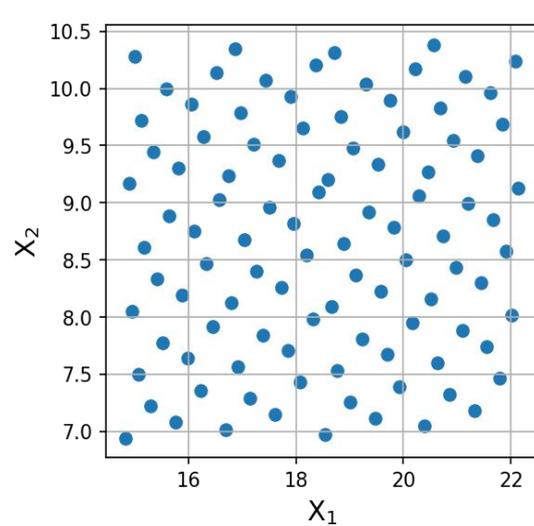
Latin Hypercube sampling (LHS)

Latin Hypercube a stratified sampling method. It divides the parameter space into equally sized bins and randomly selects samples within each bin.



Hammersley sequence sampling (HSS)

Hammersley sampling is a quasi-random sampling technique used to generate low-discrepancy sequences of points in a hypercube.



Training Dataset for Global Signal and Foreground

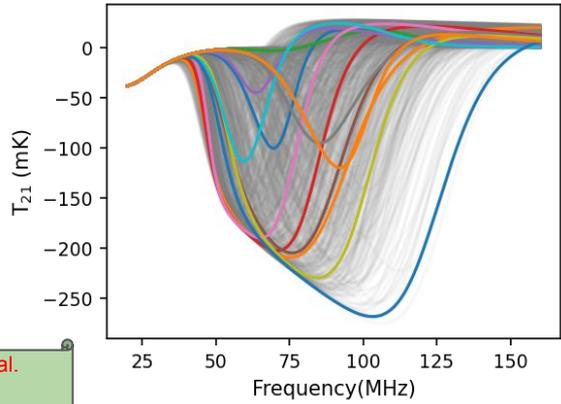
| | Parametrized | |
|------------|--------------|----------------|
| | Parameters | Ranges |
| Signal | J_{z0} | 9.27, 27.81 |
| | X_{z0} | 4.34, 13.02 |
| | T_{z0} | 4.89, 14.65 |
| | J_{dz} | 1.65, 4.96 |
| | T_{dz} | 1.41, 4.23 |
| | X_{dz} | 1.42, 4.25 |
| Foreground | a_0 | 2.97, 3.64 |
| | a_1 | -2.45, -2.37 |
| | a_2 | -0.082, -0.079 |
| | a_3 | 0.027, 0.030 |

Harker et al. 2016b

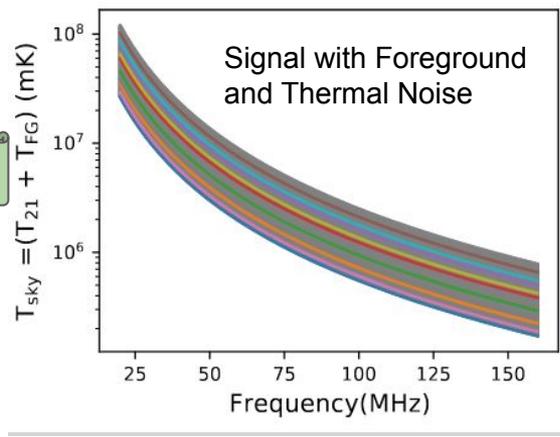
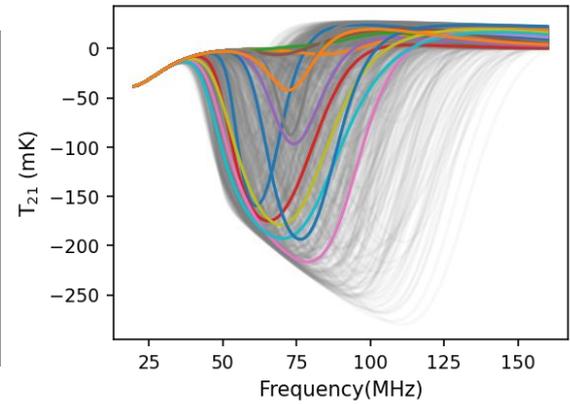
Harker et al. 2015

Tab: The range of parameters used to build the training dataset.

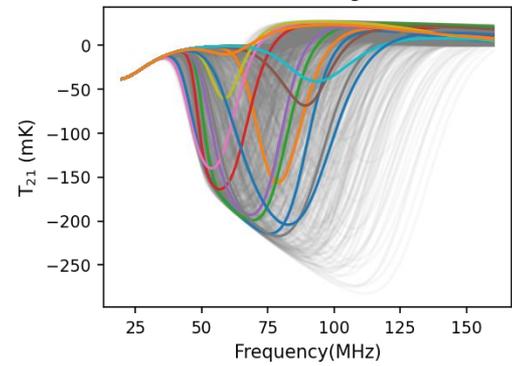
Training Datasets via Random Sampling



LHS-based Training Dataset



HSS-based Training Dataset



ANN Model Prediction

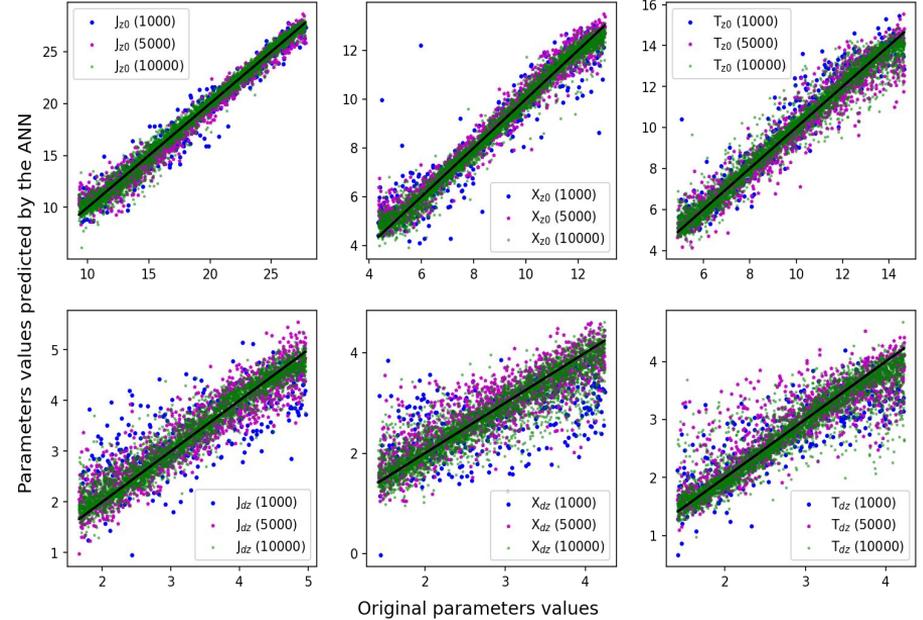
Signal Only (R^2 Score)

| Size | 1000 | | | 5000 | | | 10000 | | |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | HSS | LHS | Rand | HSS | LHS | Rand | HSS | LHS | Rand |
| Avg. | 0.6744 | 0.6594 | 0.6379 | 0.9059 | 0.8937 | 0.8837 | 0.9259 | 0.9187 | 0.9210 |
| J_{z0} | 0.9471 | 0.9442 | 0.9389 | 0.9821 | 0.9780 | 0.9770 | 0.9868 | 0.9844 | 0.9859 |
| X_{z0} | 0.8797 | 0.8552 | 0.8687 | 0.9716 | 0.9681 | 0.9693 | 0.9751 | 0.9793 | 0.9758 |
| T_{z0} | 0.8589 | 0.8899 | 0.8908 | 0.9449 | 0.9431 | 0.9410 | 0.9686 | 0.9583 | 0.9615 |
| J_{dz} | 0.5514 | 0.4943 | 0.4107 | 0.9094 | 0.8790 | 0.8978 | 0.9323 | 0.9294 | 0.9258 |
| X_{dz} | 0.1942 | 0.1372 | 0.0807 | 0.7982 | 0.7759 | 0.7200 | 0.8395 | 0.8075 | 0.8161 |
| T_{dz} | 0.6151 | 0.6377 | 0.6377 | 0.8289 | 0.8180 | 0.7973 | 0.8450 | 0.8534 | 0.8607 |

Signal with Foreground and Thermal Noise (R^2 Score)

| Size | 10000 | | | 50000 | | | 200000 | | |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | HSS | LHS | Rand | HSS | LHS | Rand | HSS | LHS | Rand |
| Total | 0.7252 | 0.7534 | 0.7307 | 0.8673 | 0.8679 | 0.8738 | 0.9139 | 0.9296 | 0.9016 |
| J_{z0} | 0.8993 | 0.9020 | 0.9109 | 0.9573 | 0.9659 | 0.9626 | 0.9753 | 0.9798 | 0.9784 |
| X_{z0} | 0.8297 | 0.8962 | 0.8414 | 0.9447 | 0.9408 | 0.9447 | 0.9798 | 0.9796 | 0.9461 |
| T_{z0} | 0.8875 | 0.8689 | 0.8634 | 0.9315 | 0.9336 | 0.9396 | 0.9483 | 0.9554 | 0.9434 |
| J_{dz} | 0.2186 | 0.2677 | 0.2813 | 0.6651 | 0.7356 | 0.7244 | 0.8246 | 0.8603 | 0.8328 |
| X_{dz} | 0.6568 | 0.6968 | 0.6595 | 0.7838 | 0.7934 | 0.8025 | 0.8248 | 0.8458 | 0.8361 |
| T_{dz} | 0.4979 | 0.5837 | 0.5005 | 0.7953 | 0.7472 | 0.7875 | 0.8896 | 0.9083 | 0.7240 |
| a_0 | 0.7229 | 0.7452 | 0.7292 | 0.8375 | 0.8672 | 0.8773 | 0.9086 | 0.9214 | 0.8889 |
| a_1 | 0.9994 | 0.9997 | 0.9996 | 0.9989 | 0.9997 | 0.9999 | 0.9994 | 0.9999 | 0.9994 |
| a_2 | 0.6095 | 0.6460 | 0.5964 | 0.8274 | 0.7659 | 0.7684 | 0.8606 | 0.9152 | 0.9363 |
| a_3 | 0.9298 | 0.9281 | 0.9248 | 0.9319 | 0.9296 | 0.9311 | 0.9277 | 0.9302 | 0.9304 |

Signal Parameters Predicted by the ANN



ANN Model Prediction

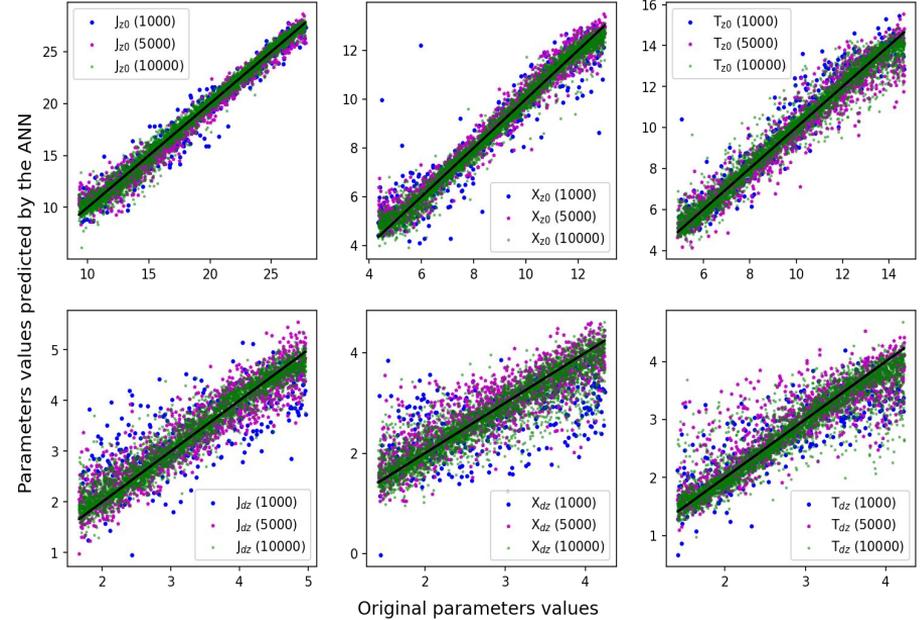
Signal Only (R^2 Score)

| Size | 1000 | | | HSS | 5000 | | | 10000 | | |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
| | HSS | LHS | Rand | | HSS | LHS | Rand | HSS | LHS | Rand |
| Avg. | 0.6744 | 0.6594 | 0.6379 | 0.9059 | 0.8937 | 0.8837 | 0.9259 | 0.9187 | 0.9210 | |
| J_{z0} | 0.9471 | 0.9442 | 0.9389 | 0.9821 | 0.9780 | 0.9770 | 0.9868 | 0.9844 | 0.9859 | |
| X_{z0} | 0.8797 | 0.8552 | 0.8687 | 0.9716 | 0.9681 | 0.9693 | 0.9751 | 0.9793 | 0.9758 | |
| T_{z0} | 0.8589 | 0.8899 | 0.8908 | 0.9449 | 0.9431 | 0.9410 | 0.9686 | 0.9583 | 0.9615 | |
| J_{dz} | 0.5514 | 0.4943 | 0.4107 | 0.9094 | 0.8790 | 0.8978 | 0.9323 | 0.9294 | 0.9258 | |
| X_{dz} | 0.1942 | 0.1372 | 0.0807 | 0.7982 | 0.7759 | 0.7200 | 0.8395 | 0.8075 | 0.8161 | |
| T_{dz} | 0.6151 | 0.6377 | 0.6377 | 0.8289 | 0.8180 | 0.7973 | 0.8450 | 0.8534 | 0.8607 | |

Signal with Foreground and Thermal Noise (R^2 Score)

| Size | 10000 | | | HSS | 50000 | | | 200000 | | |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
| | HSS | LHS | Rand | | HSS | LHS | Rand | HSS | LHS | Rand |
| Total | 0.7252 | 0.7534 | 0.7307 | 0.8673 | 0.8679 | 0.8738 | 0.9139 | 0.9296 | 0.9016 | |
| J_{z0} | 0.8993 | 0.9020 | 0.9109 | 0.9573 | 0.9659 | 0.9626 | 0.9753 | 0.9798 | 0.9784 | |
| X_{z0} | 0.8297 | 0.8962 | 0.8414 | 0.9447 | 0.9408 | 0.9447 | 0.9798 | 0.9796 | 0.9461 | |
| T_{z0} | 0.8875 | 0.8689 | 0.8634 | 0.9315 | 0.9336 | 0.9396 | 0.9483 | 0.9554 | 0.9434 | |
| J_{dz} | 0.2186 | 0.2677 | 0.2813 | 0.6651 | 0.7356 | 0.7244 | 0.8246 | 0.8603 | 0.8328 | |
| X_{dz} | 0.6568 | 0.6968 | 0.6595 | 0.7838 | 0.7934 | 0.8025 | 0.8248 | 0.8458 | 0.8361 | |
| T_{dz} | 0.4979 | 0.5837 | 0.5005 | 0.7953 | 0.7472 | 0.7875 | 0.8896 | 0.9083 | 0.7240 | |
| a_0 | 0.7229 | 0.7452 | 0.7292 | 0.8375 | 0.8672 | 0.8773 | 0.9086 | 0.9214 | 0.8889 | |
| a_1 | 0.9994 | 0.9997 | 0.9996 | 0.9989 | 0.9997 | 0.9999 | 0.9994 | 0.9999 | 0.9994 | |
| a_2 | 0.6095 | 0.6460 | 0.5964 | 0.8274 | 0.7659 | 0.7684 | 0.8606 | 0.9152 | 0.9363 | |
| a_3 | 0.9298 | 0.9281 | 0.9248 | 0.9319 | 0.9296 | 0.9311 | 0.9277 | 0.9302 | 0.9304 | |

Signal Parameters Predicted by the ANN



ANN Model Prediction

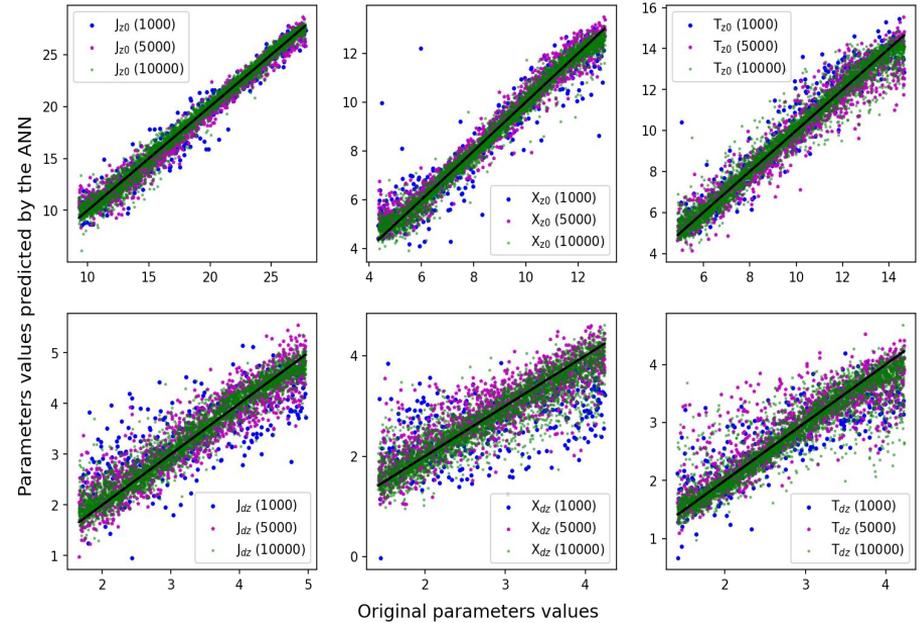
Signal Only (R^2 Score)

| Size | 1000 | | | HSS | 5000 | | | 10000 | | |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
| | HSS | LHS | Rand | | HSS | LHS | Rand | HSS | LHS | Rand |
| Avg. | 0.6744 | 0.6594 | 0.6379 | 0.9059 | 0.8937 | 0.8837 | 0.9259 | 0.9187 | 0.9210 | |
| J_{z0} | 0.9471 | 0.9442 | 0.9389 | 0.9821 | 0.9780 | 0.9770 | 0.9868 | 0.9844 | 0.9859 | |
| X_{z0} | 0.8797 | 0.8552 | 0.8687 | 0.9716 | 0.9681 | 0.9693 | 0.9751 | 0.9793 | 0.9758 | |
| T_{z0} | 0.8589 | 0.8899 | 0.8908 | 0.9449 | 0.9431 | 0.9410 | 0.9686 | 0.9583 | 0.9615 | |
| J_{dz} | 0.5514 | 0.4943 | 0.4107 | 0.9094 | 0.8790 | 0.8978 | 0.9323 | 0.9294 | 0.9258 | |
| X_{dz} | 0.1942 | 0.1372 | 0.0807 | 0.7982 | 0.7759 | 0.7200 | 0.8395 | 0.8075 | 0.8161 | |
| T_{dz} | 0.6151 | 0.6377 | 0.6377 | 0.8289 | 0.8180 | 0.7973 | 0.8450 | 0.8534 | 0.8607 | |

Signal with Foreground and Thermal Noise (R^2 Score)

| Size | 10000 | | | HSS | 50000 | | | 200000 | | |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
| | HSS | LHS | Rand | | HSS | LHS | Rand | HSS | LHS | Rand |
| Total | 0.7252 | 0.7534 | 0.7307 | 0.8673 | 0.8679 | 0.8738 | 0.9139 | 0.9296 | 0.9016 | |
| J_{z0} | 0.8993 | 0.9020 | 0.9109 | 0.9573 | 0.9659 | 0.9626 | 0.9753 | 0.9798 | 0.9784 | |
| X_{z0} | 0.8297 | 0.8962 | 0.8414 | 0.9447 | 0.9408 | 0.9447 | 0.9798 | 0.9796 | 0.9461 | |
| T_{z0} | 0.8875 | 0.8689 | 0.8634 | 0.9315 | 0.9336 | 0.9396 | 0.9483 | 0.9554 | 0.9434 | |
| J_{dz} | 0.2186 | 0.2677 | 0.2813 | 0.6651 | 0.7356 | 0.7244 | 0.8246 | 0.8603 | 0.8328 | |
| X_{dz} | 0.6568 | 0.6968 | 0.6595 | 0.7838 | 0.7934 | 0.8025 | 0.8248 | 0.8458 | 0.8361 | |
| T_{dz} | 0.4979 | 0.5837 | 0.5005 | 0.7953 | 0.7472 | 0.7875 | 0.8896 | 0.9083 | 0.7240 | |
| a_0 | 0.7229 | 0.7452 | 0.7292 | 0.8375 | 0.8672 | 0.8773 | 0.9086 | 0.9214 | 0.8889 | |
| a_1 | 0.9994 | 0.9997 | 0.9996 | 0.9989 | 0.9997 | 0.9999 | 0.9994 | 0.9999 | 0.9994 | |
| a_2 | 0.6095 | 0.6460 | 0.5964 | 0.8274 | 0.7659 | 0.7684 | 0.8606 | 0.9152 | 0.9363 | |
| a_3 | 0.9298 | 0.9281 | 0.9248 | 0.9319 | 0.9296 | 0.9311 | 0.9277 | 0.9302 | 0.9304 | |

Signal Parameters Predicted by the ANN



- 1) In the lower sample size Hammersley perform slightly better in the lower dimensions.
- 2) At large number of sample size all sampling methods conserved at the same point.

Generalizability Test of the ANN Models

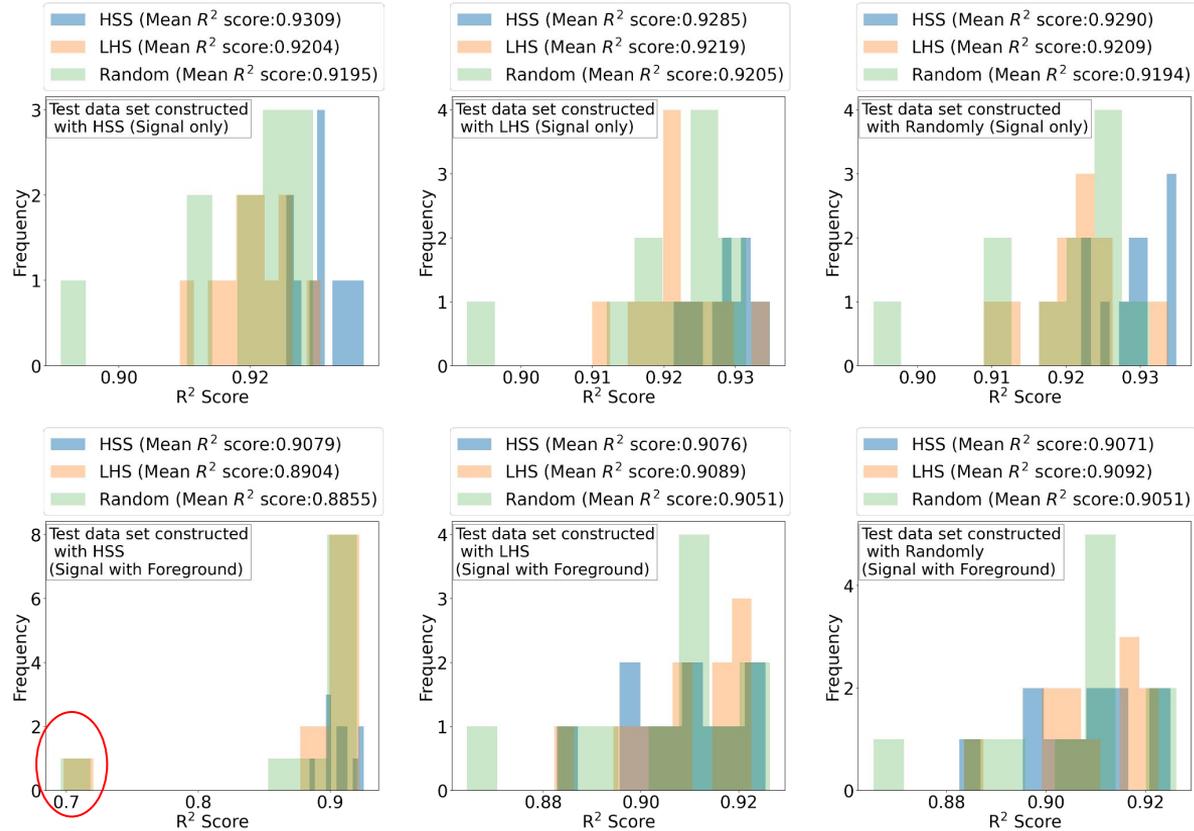


Fig : This shows the ANN model predictions for various trials trained on datasets sampled using HSS, LHS, and Random Sampling methods with optimal sample sizes. Histograms depict ANN prediction accuracy measured in terms of R^2 scores.

Generalizability Test of the ANN Models

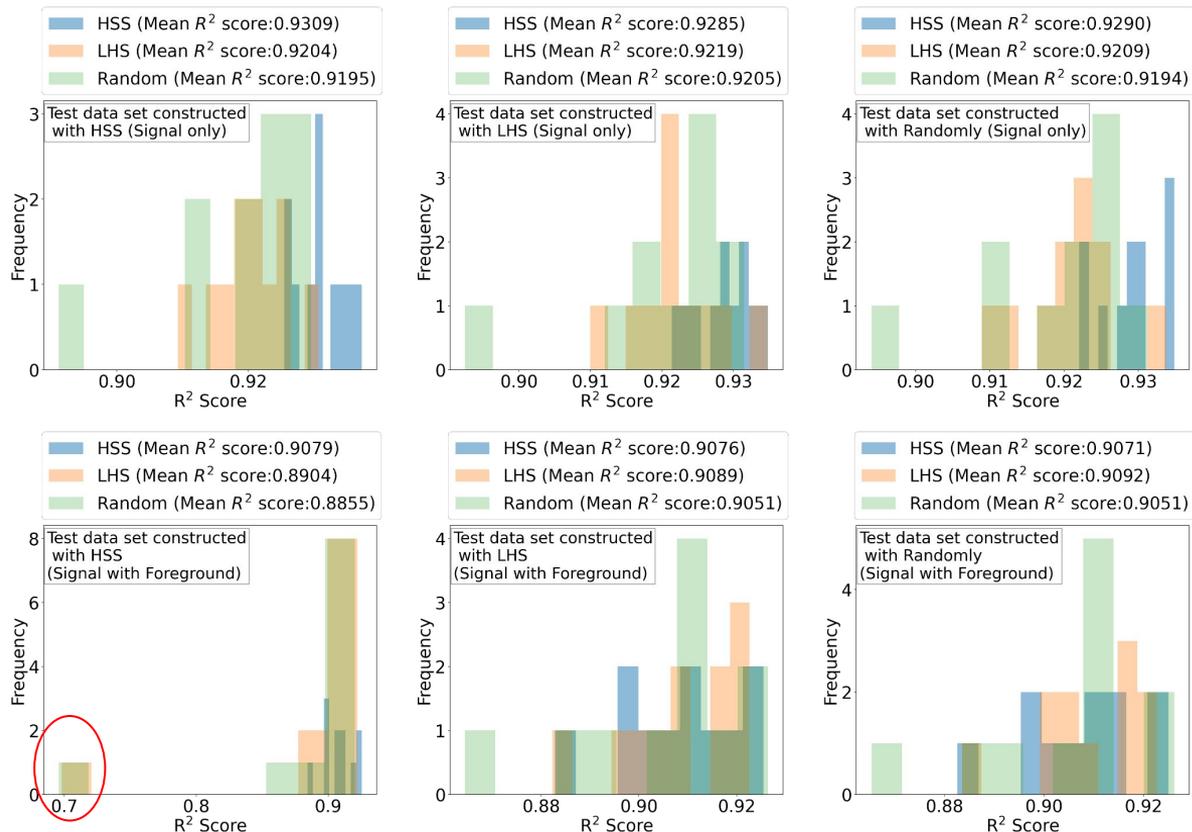


Fig : This shows the ANN model predictions for various trials trained on datasets sampled using HSS, LHS, and Random Sampling methods with optimal sample sizes. Histograms depict ANN prediction accuracy measured in terms of R² scores.

Takeaway & Summary

- Achieving a high level of accuracy in training the ANN model necessitates an ample number of samples, regardless of the sampling technique employed.
- The number of drawing of the samples highly dependent on the dimensionality and complexity.
- Among the given sampling techniques we found the model train with Hammersley Sequence are more robust.

ANN Model Prediction with different Signal Model

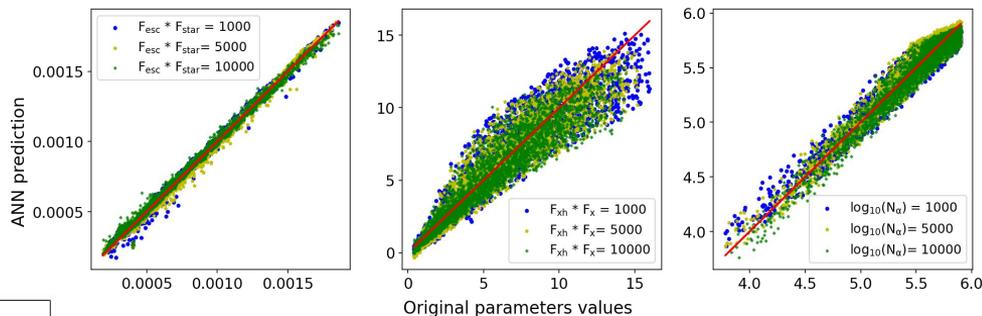
Signal Only (R^2 Score)

| Size | 1000 | | | 5000 | | | 10000 | | |
|---------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | HSS | LHS | Rand | HSS | LHS | Rand | HSS | LHS | Rand |
| Avg. | 0.9155 | 0.9252 | 0.9214 | 0.9392 | 0.9367 | 0.9346 | 0.9447 | 0.9446 | 0.9418 |
| $f_* \cdot f_{esc}$ | 0.9936 | 0.9922 | 0.9893 | 0.9945 | 0.9957 | 0.9940 | 0.9974 | 0.9970 | 0.9944 |
| $f_{X,h} \cdot f_X$ | 0.8255 | 0.8373 | 0.8408 | 0.8785 | 0.8682 | 0.8642 | 0.8837 | 0.8737 | 0.8783 |
| N_α | 0.9275 | 0.9454 | 0.9341 | 0.9531 | 0.9539 | 0.9555 | 0.9531 | 0.9532 | 0.9527 |

Signal with Foreground and Thermal Noise (R^2 Score)

| Size | 10000 | | | 50000 | | | 100000 | | |
|---------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | HSS | LHS | Rand | HSS | LHS | Rand | HSS | LHS | Rand |
| Avg. | 0.9277 | 0.9187 | 0.9018 | 0.9505 | 0.9448 | 0.9429 | 0.9670 | 0.9647 | 0.9608 |
| $f_* \cdot f_{esc}$ | 0.9813 | 0.9532 | 0.9776 | 0.9890 | 0.9510 | 0.9634 | 0.9843 | 0.9903 | 0.9860 |
| $f_{X,h} \cdot f_X$ | 0.8574 | 0.8727 | 0.8272 | 0.8807 | 0.8502 | 0.8570 | 0.9139 | 0.9168 | 0.9142 |
| N_α | 0.9395 | 0.9398 | 0.8502 | 0.9512 | 0.9454 | 0.9405 | 0.9620 | 0.9563 | 0.9629 |
| a_0 | 0.9925 | 0.9978 | 0.9960 | 0.9968 | 0.9959 | 0.9969 | 0.9988 | 0.9986 | 0.9973 |
| a_1 | 0.9996 | 0.9969 | 0.9974 | 0.9993 | 0.9982 | 0.9998 | 0.9999 | 0.9991 | 0.9998 |
| a_2 | 0.7921 | 0.7281 | 0.7291 | 0.9080 | 0.9386 | 0.9084 | 0.9590 | 0.9415 | 0.9242 |
| a_3 | 0.9317 | 0.9427 | 0.9353 | 0.9340 | 0.9327 | 0.9337 | 0.9505 | 0.9504 | 0.9400 |

Signal Parameters Predicted by the ANN



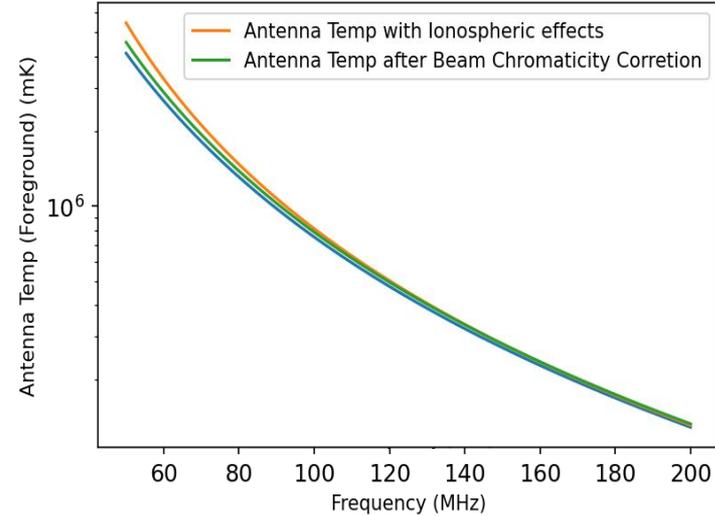
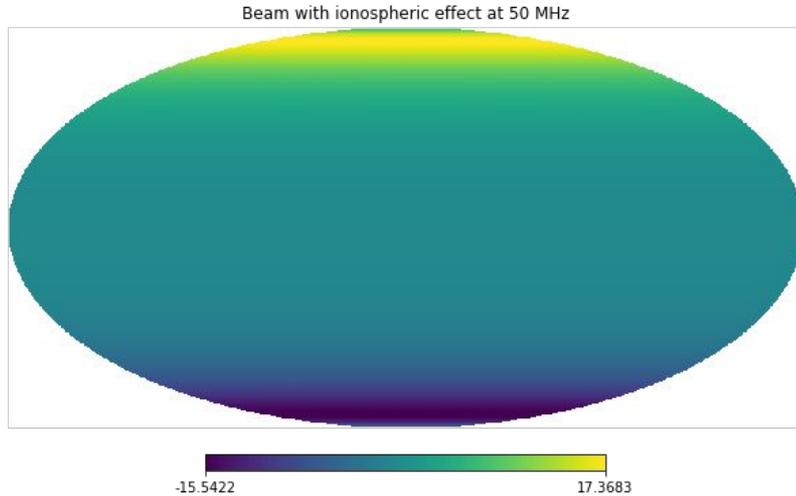
- Consistent trends in prediction accuracy were observed across different Signal Models.

Dynamic Ionospheric Effects



Ionospheric effect on Antenna Beam

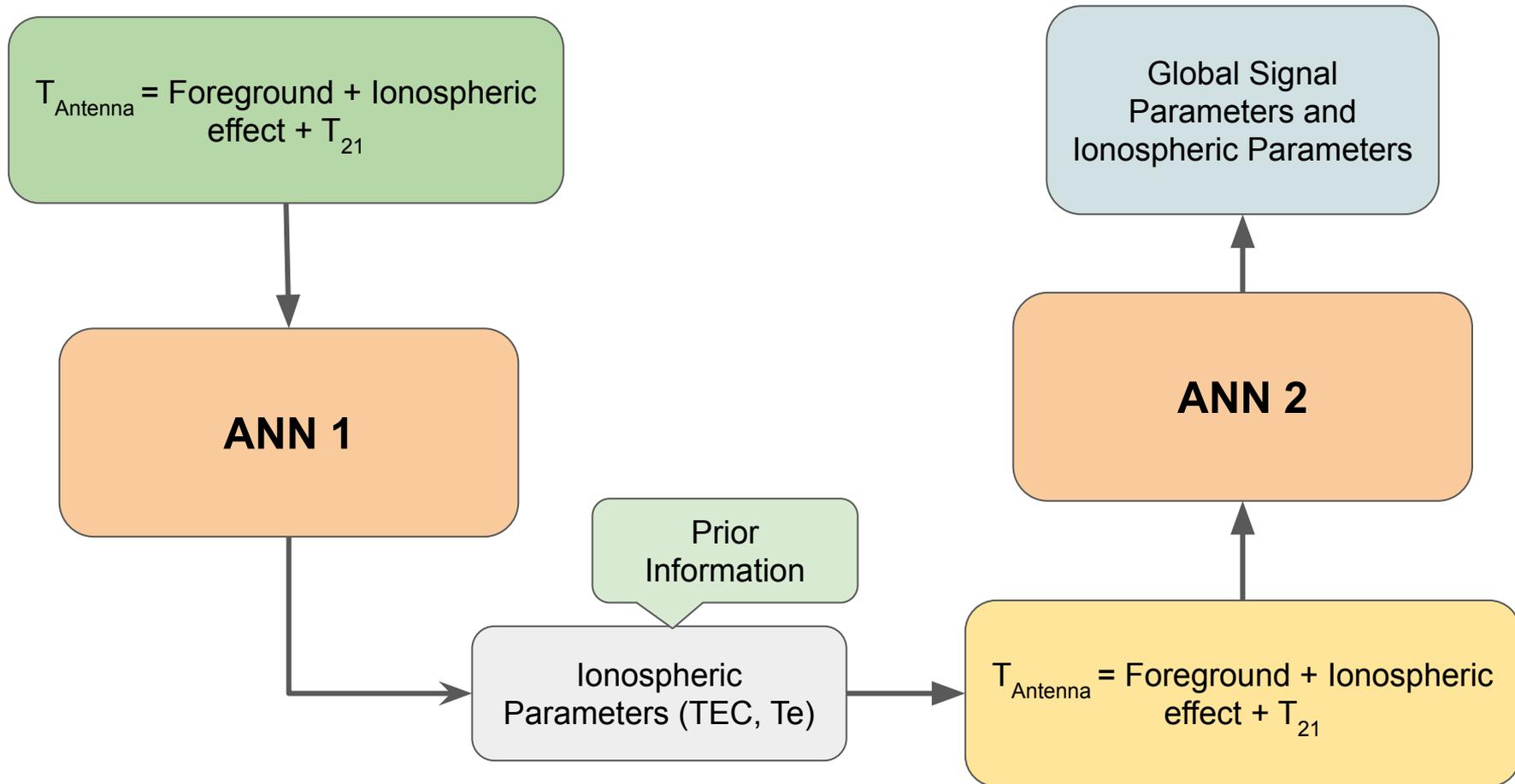
$$\hat{B}(\nu, \theta, \phi) = B(\nu, \theta - \delta\theta, \phi) \cdot L(\nu, \theta - \delta\theta, \phi)$$



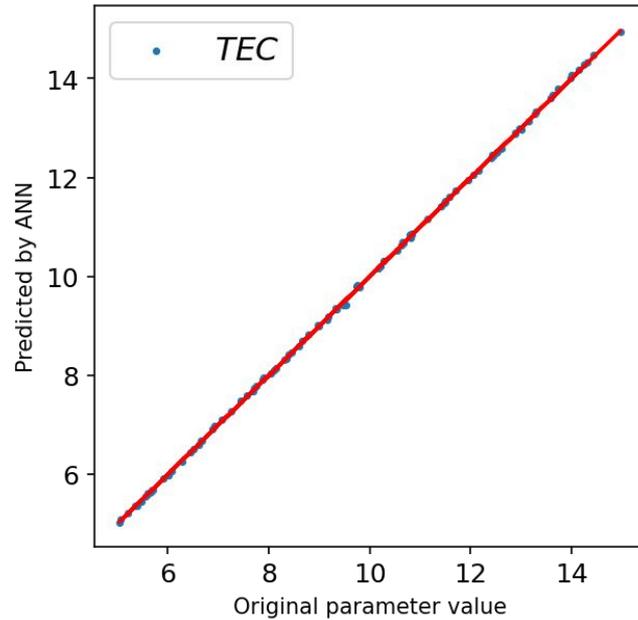
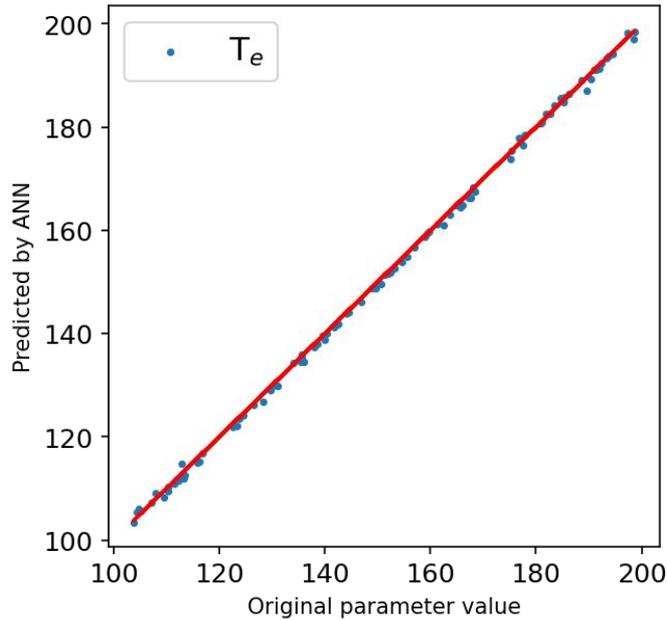
$$B_{\text{factor}}(\nu) = \frac{\int_{\Omega} T_{\text{sky}}(\nu_{150}, \Omega) B(\nu, \Omega) d\Omega}{\int_{\Omega} T_{\text{sky}}(\nu_{150}, \Omega) B(\nu_{150}, \Omega) d\Omega},$$

$$T_A(\nu) = \int_0^{2\pi} d\phi \int_0^{\pi/2} B(\nu, \theta, \phi) [T_{\text{te}}(\nu, \theta) + \mathcal{L}(\nu, \theta) \times \hat{T}_{\text{sky}}(\nu, \theta, \phi)] \sin \theta d\theta.$$

2 Steps ANNs Training

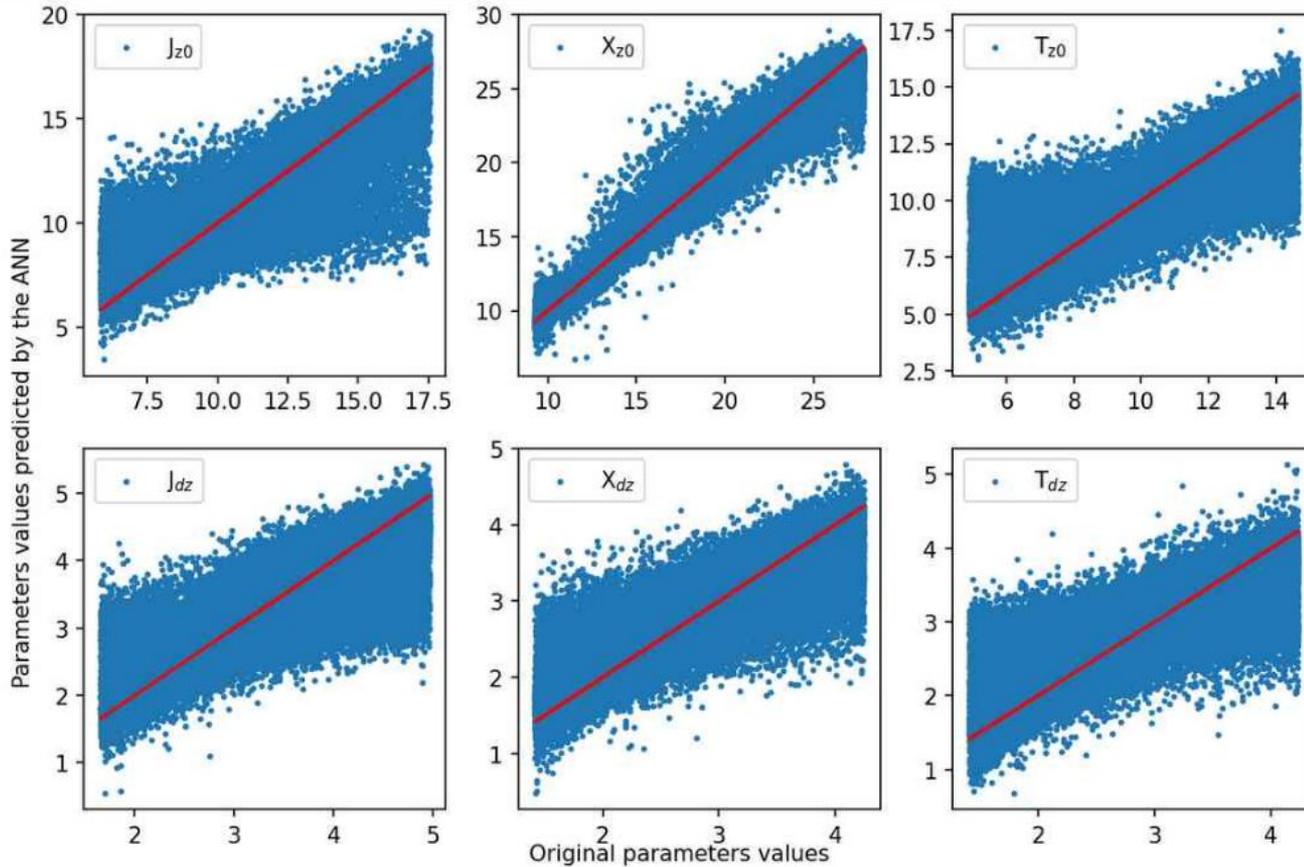


ANN prediction of Ionospheric Parameters (ANN1)



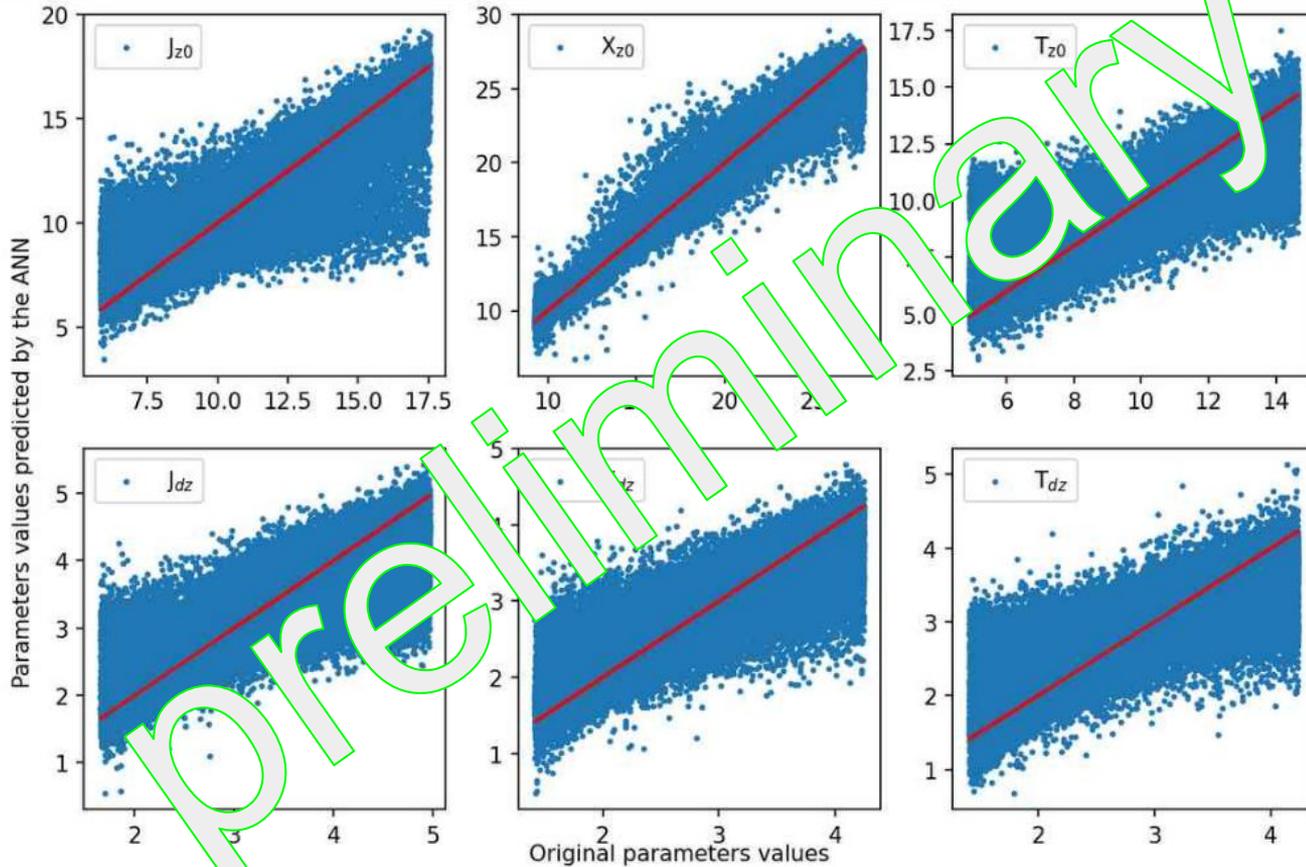
R^2 score ≈ 0.99

ANN 2 prediction

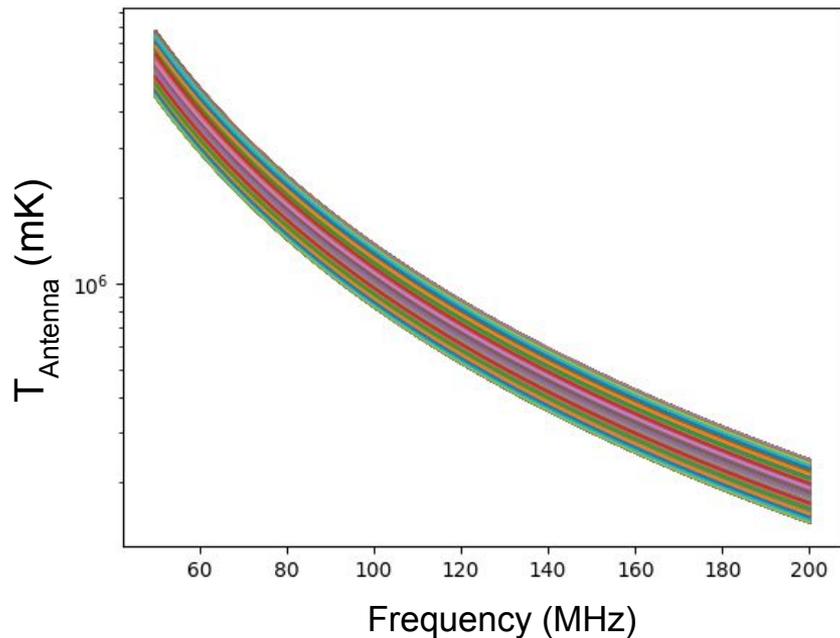


R^2 score ranging
0.60 to 0.90

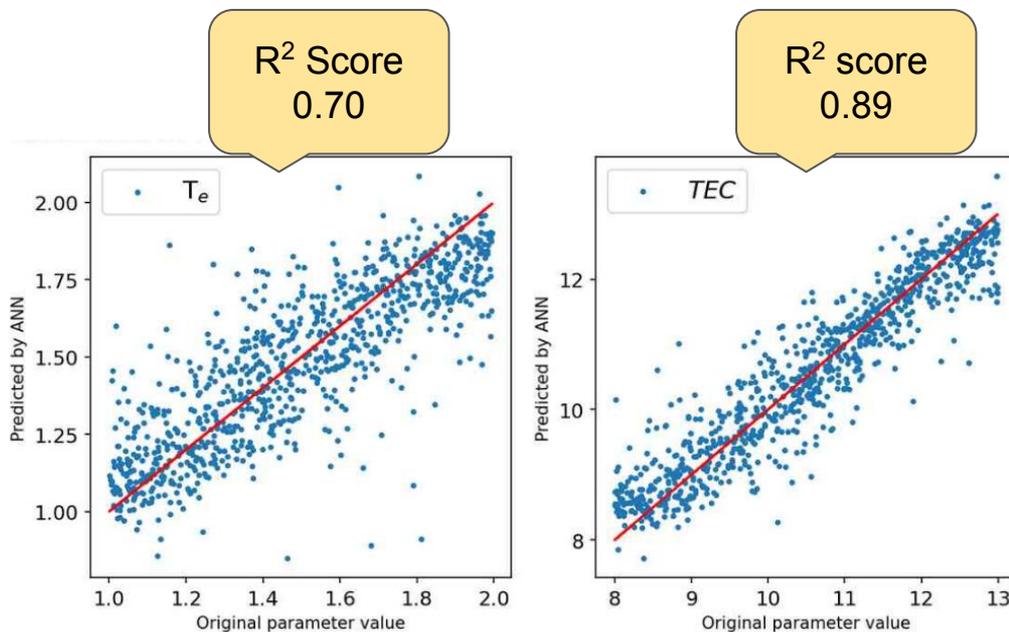
ANN 2 prediction



R² score ranging
0.60 to 0.90



Time Varying Global Sky Model with Ionospheric effects



Ongoing Work

- Further plan to study effects of Chromatic Effects of Soil and Grounding.
- Plan to compare these effects across various antenna beams to enhance our pipeline's robustness.

Thank You.

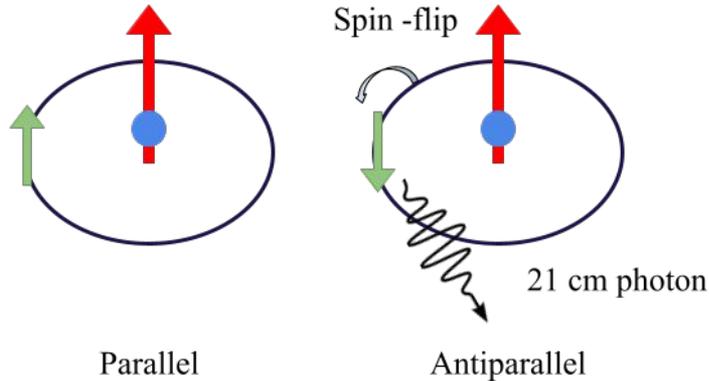
I am seeking a Postdoctoral position. If anyone is hiring, please let me know!

Email : anshumantripathi85@gmail.com



21 cm line

Arises due to “spin-flip” transition in hydrogen atom.



Triplet state
Higher energy

Singlet state
Lower energy

Spin Temp

- Intrinsically **forbidden**
- Einstein' A coefficient:
Probability of transition:
 $A_{10} = 2.85 \times 10^{-15} \text{s}$
- Lifetime: **10^7 years!**

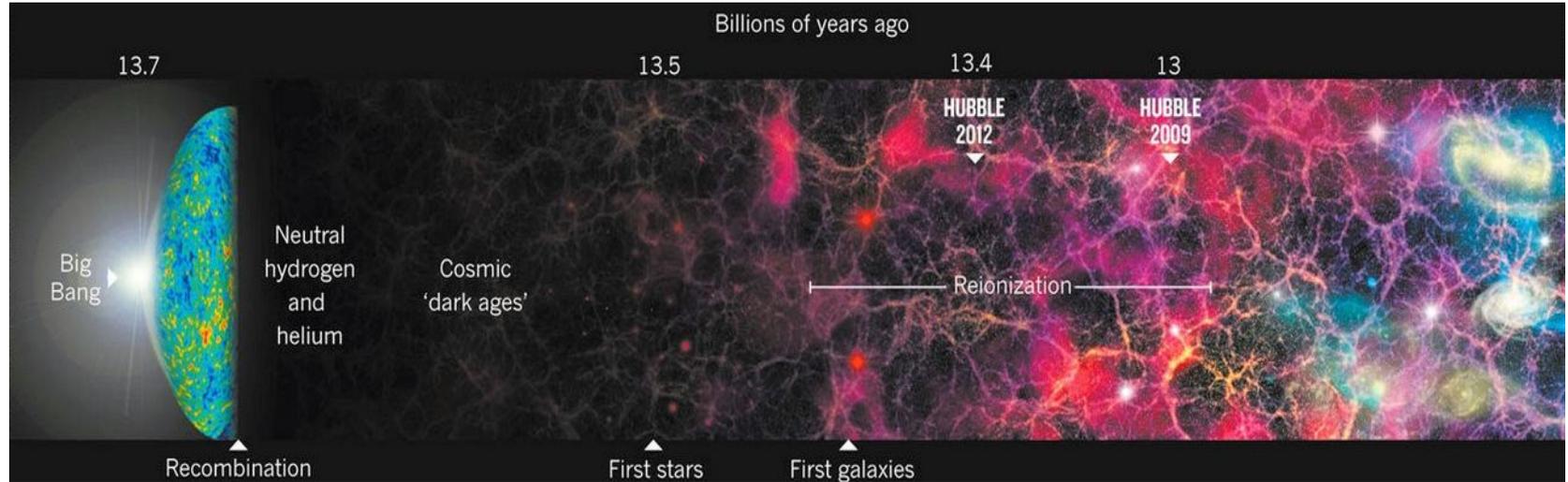
- Abundance of Hydrogen(75%)
- **Coupling:**
Collisions
Lyman α pumping
Scattering with Cosmic
microwave background(CMB)

Make it happen and seen!

- ✗ Ratio of number of H atoms in triplet to singlet state..

$$\left(\frac{n_1}{n_0} \right) = 3 \exp \left(\frac{-T_*}{T_s} \right)$$

The First Dawn



Source: <https://astrobits.org/wp-content/uploads/2015/05/cover.png>