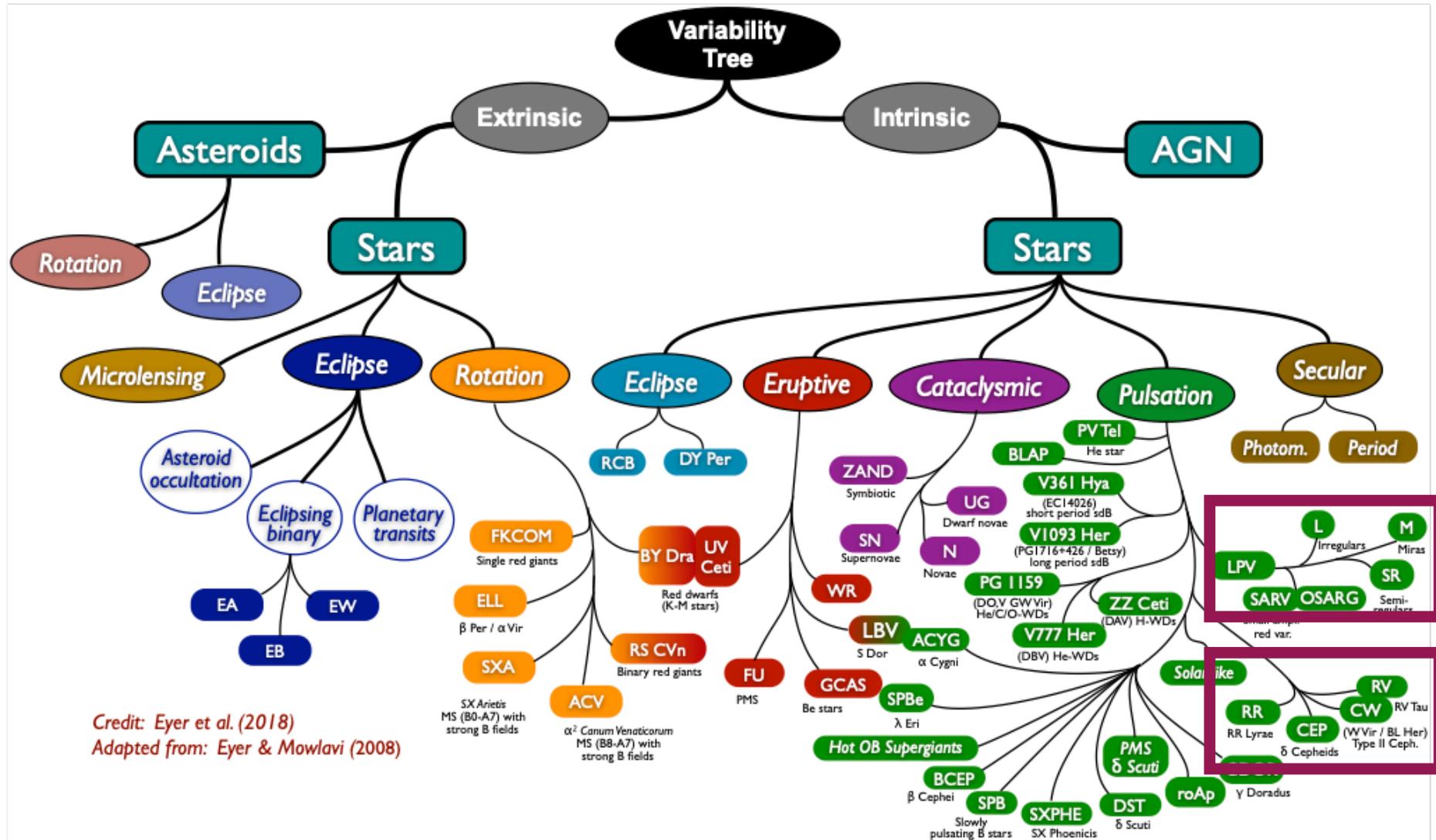


Pulsating stars and their fundamental parameters with ML

Anupam Bhardwaj (IUCAA)



Stellar variability and pulsation

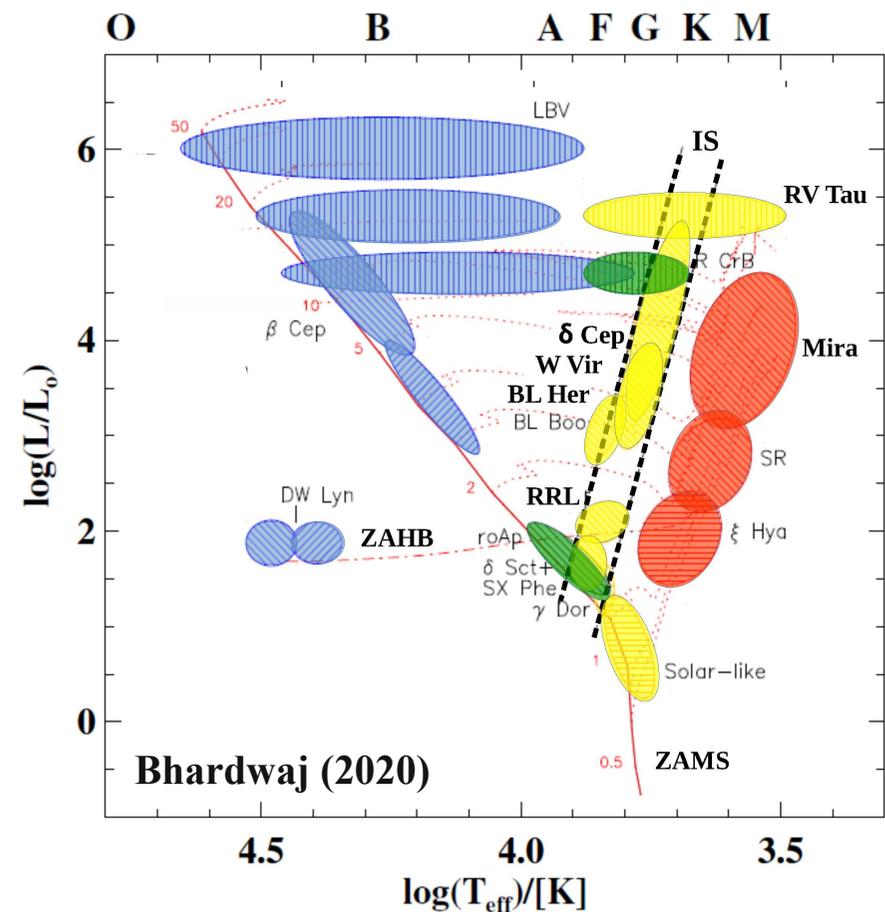
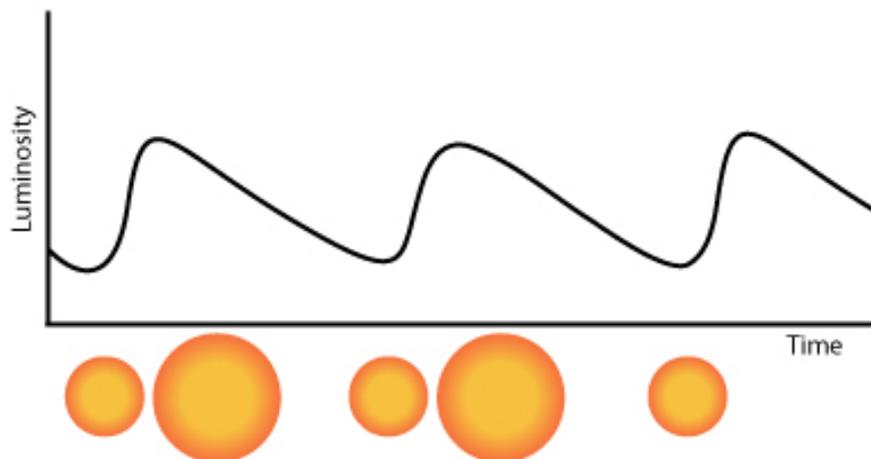


Credit: Eyer et al. (2018)
Adapted from: Eyer & Mowlavi (2008)

Pulsating variable stars

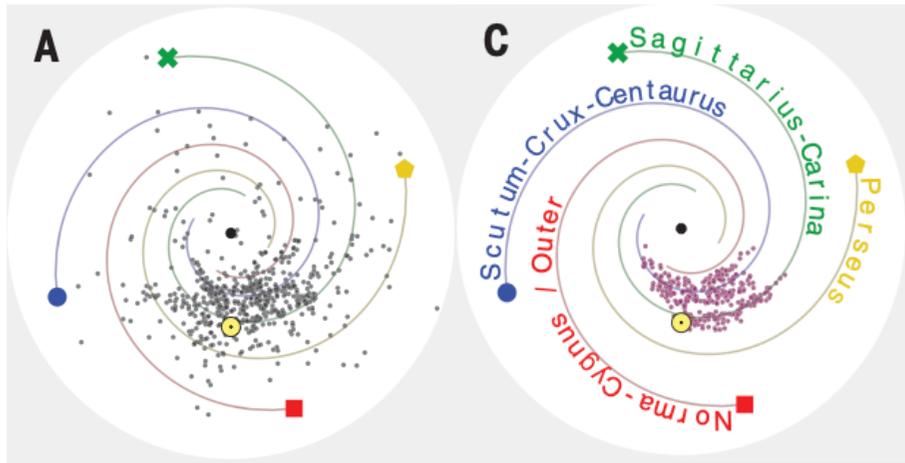
Within and beyond the “Instability Strip” in the HR diagram

- Classical or Type I Cepheids
- Type II Cepheids
- Anomalous Cepheids
- RR Lyrae variables
- Miras

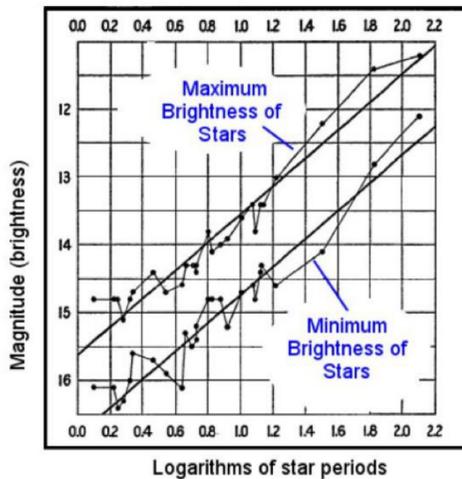
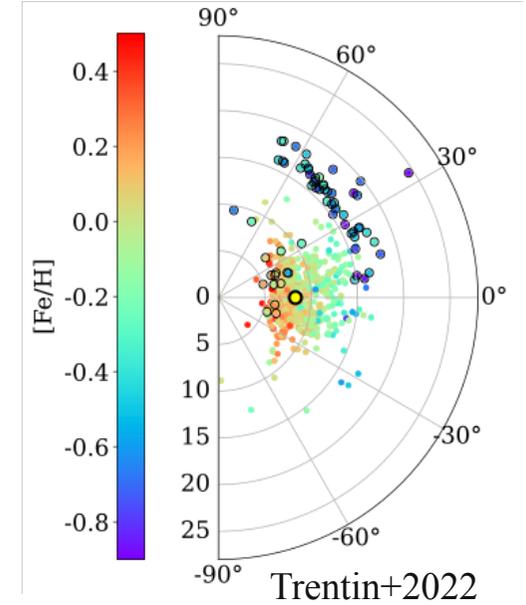
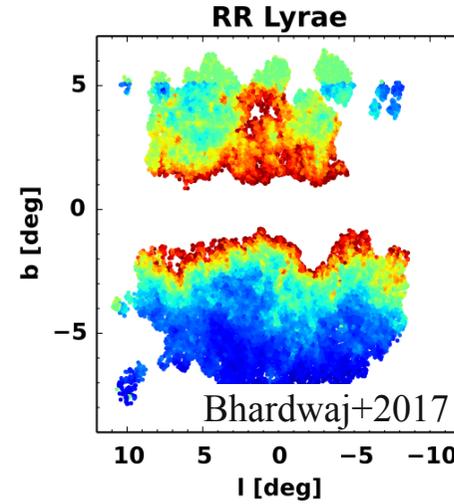


Pulsating variable stars

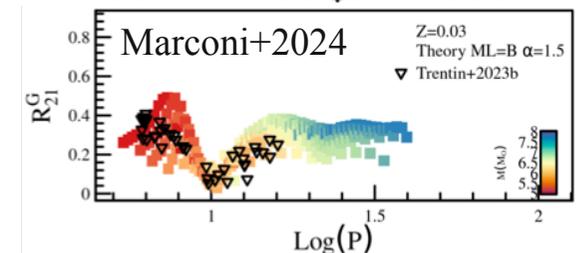
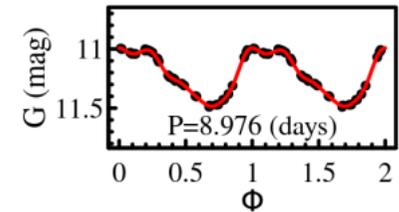
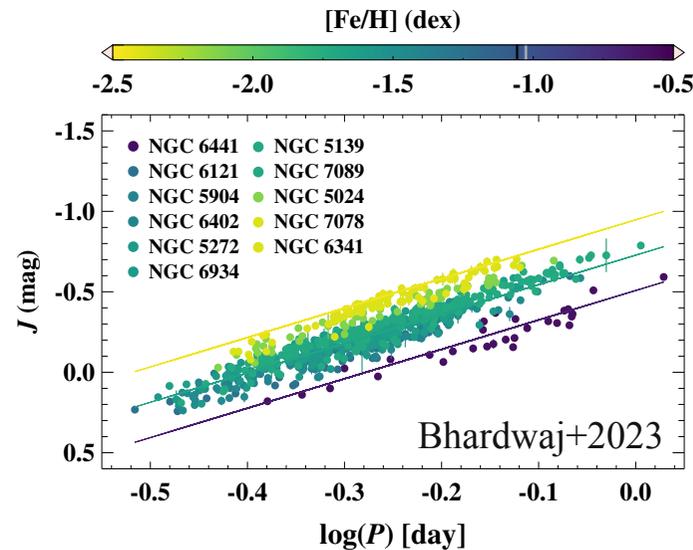
Stellar population tracers and distance indicators



Skowron+2019

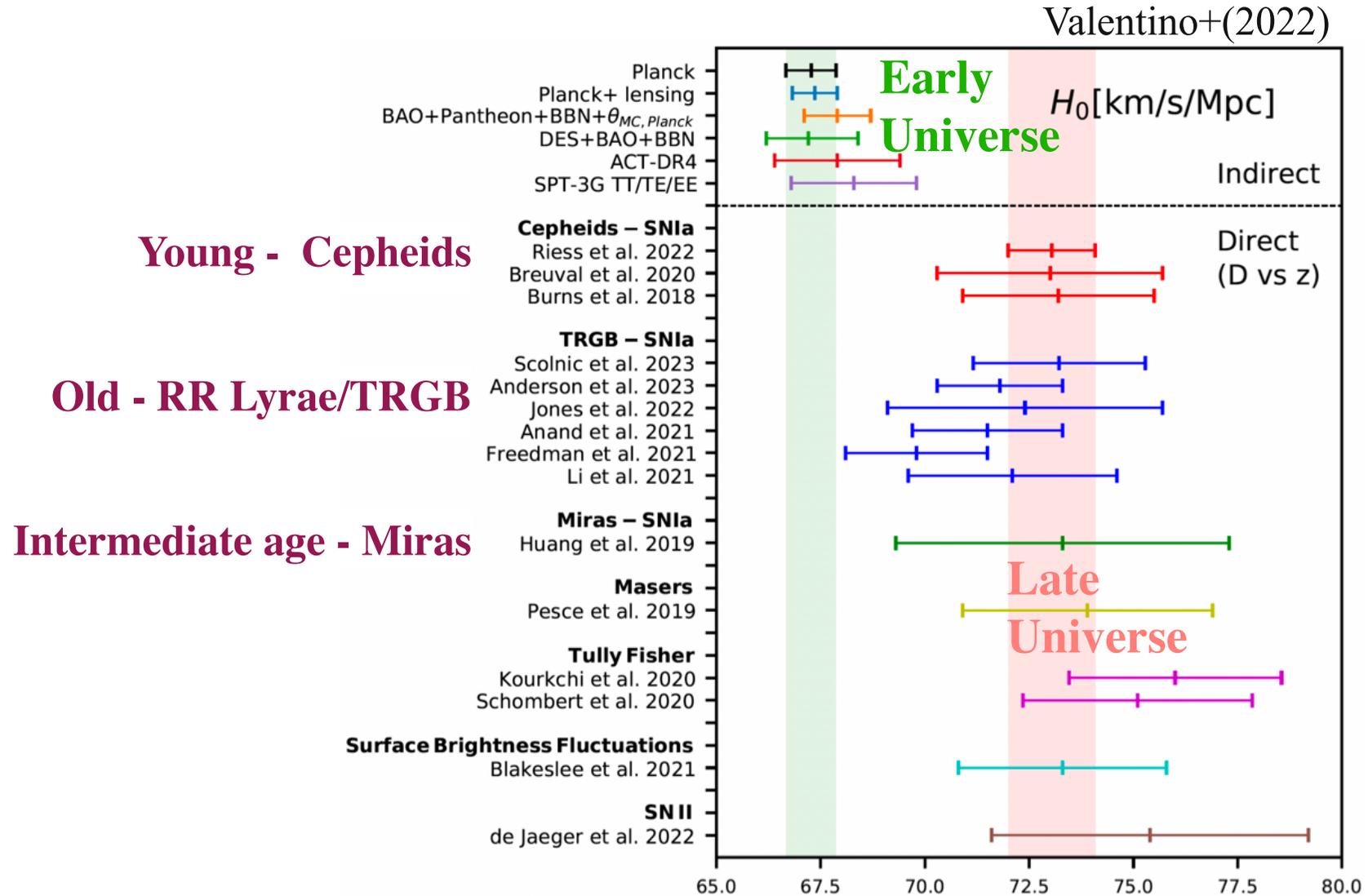


Classical Cepheids in the Magellanic Clouds
Leavitt and Pickering (1912)



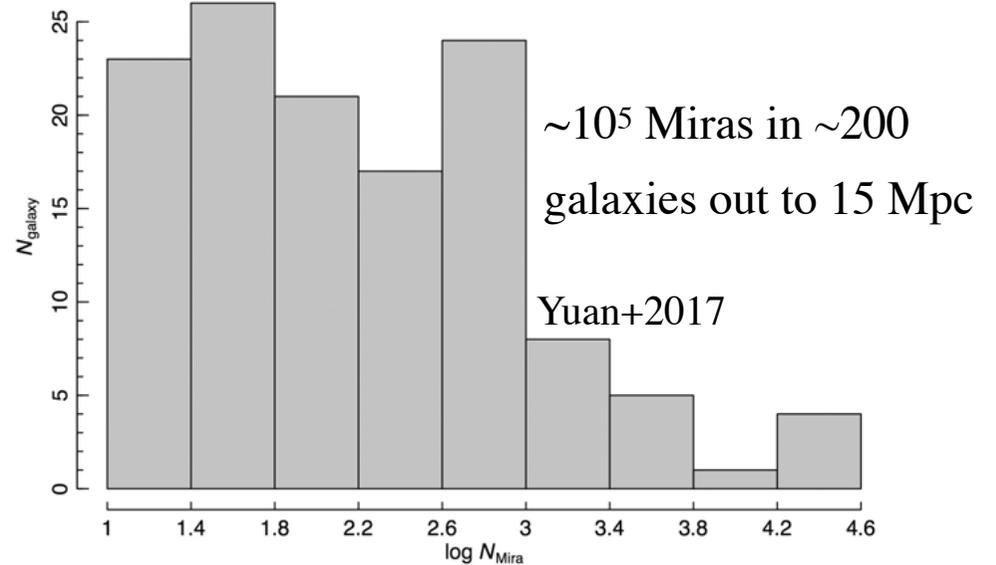
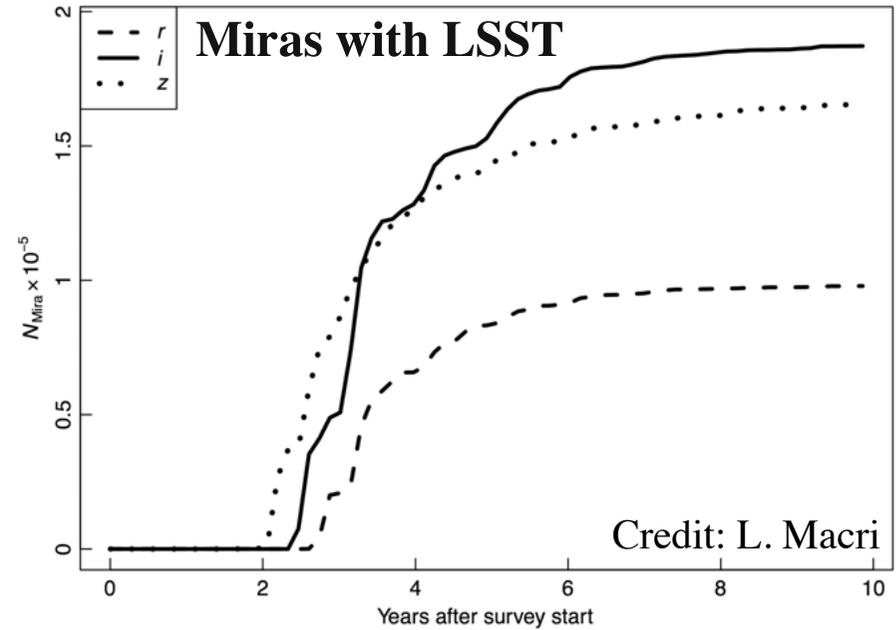
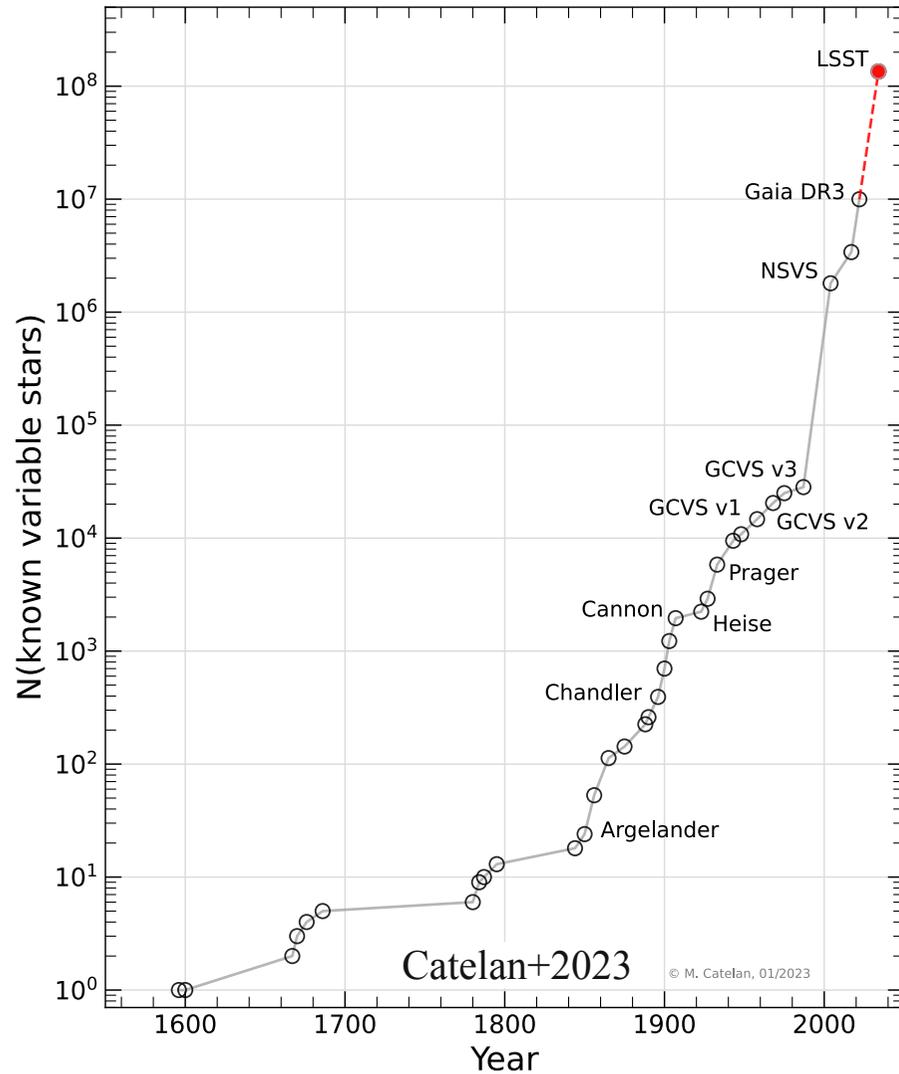
Cosmic distance ladder

Based on standard candles of different ages and metallicities



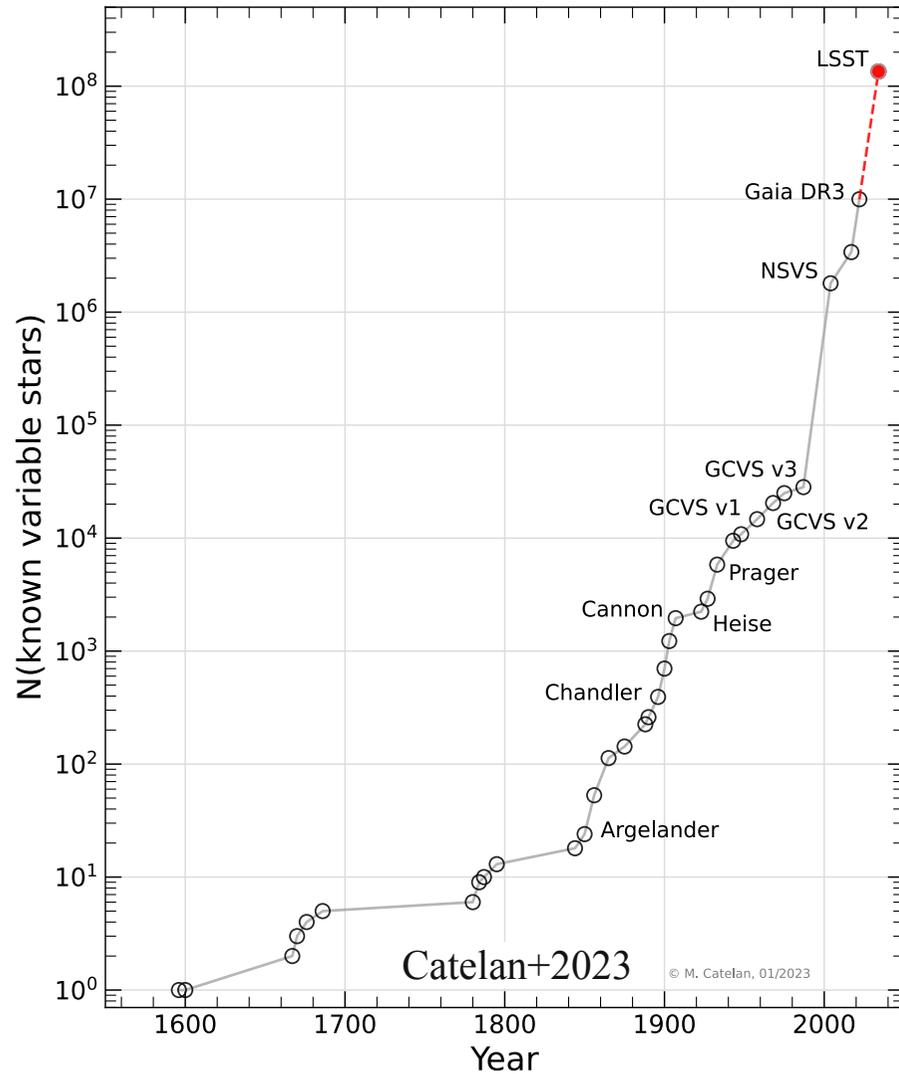
Variable star numbers

Now and with LSST...

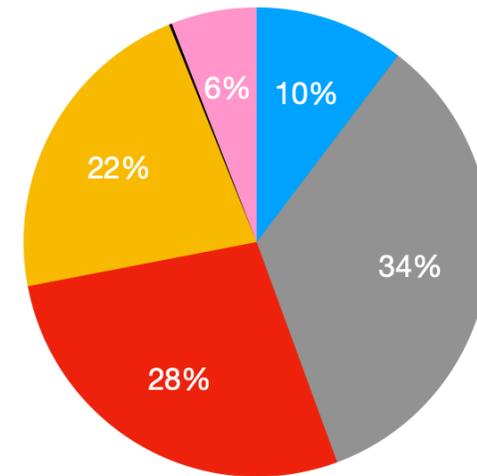


Variable star numbers

Now and with LSST...



- AGN
- Rotation (+other)
- Eruptive/Cataclysmic
- Pulsation
- Eclipsing systems
- Other

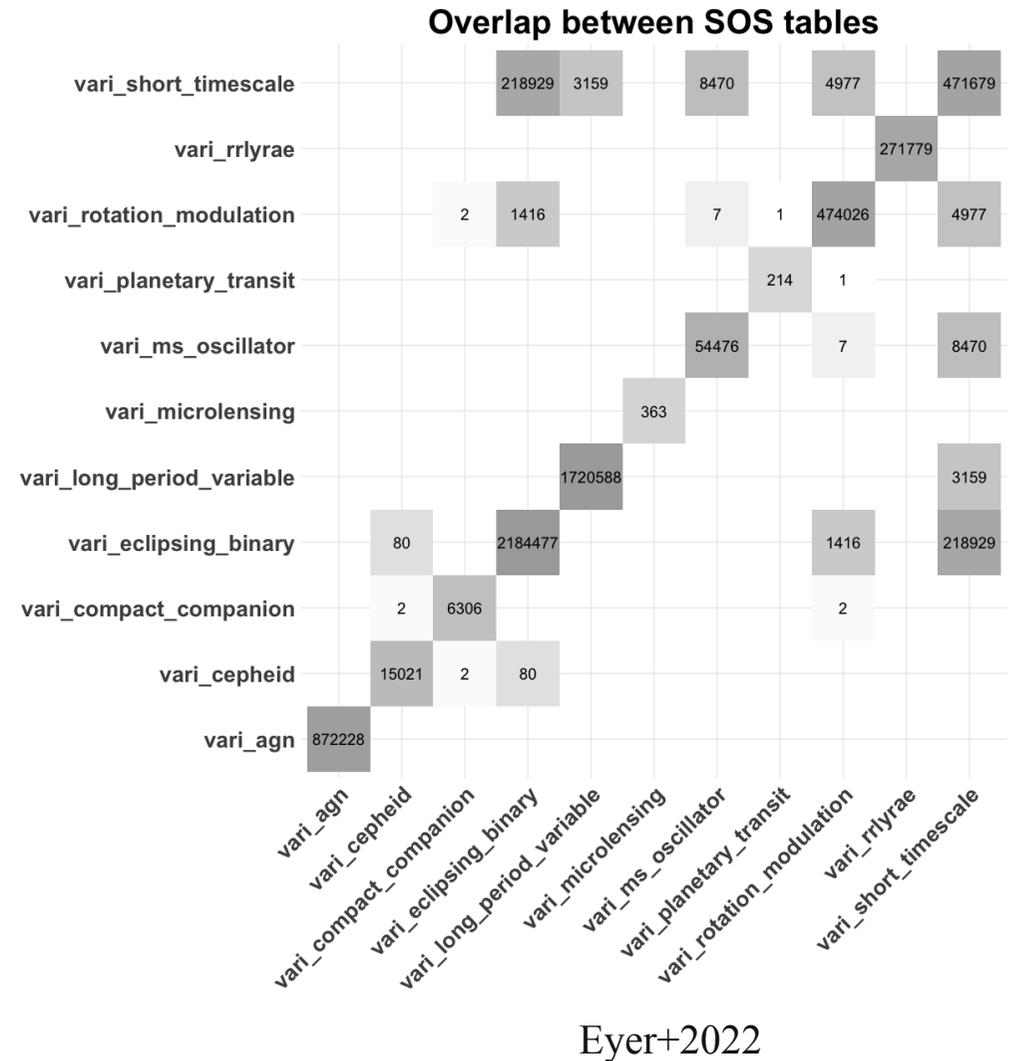
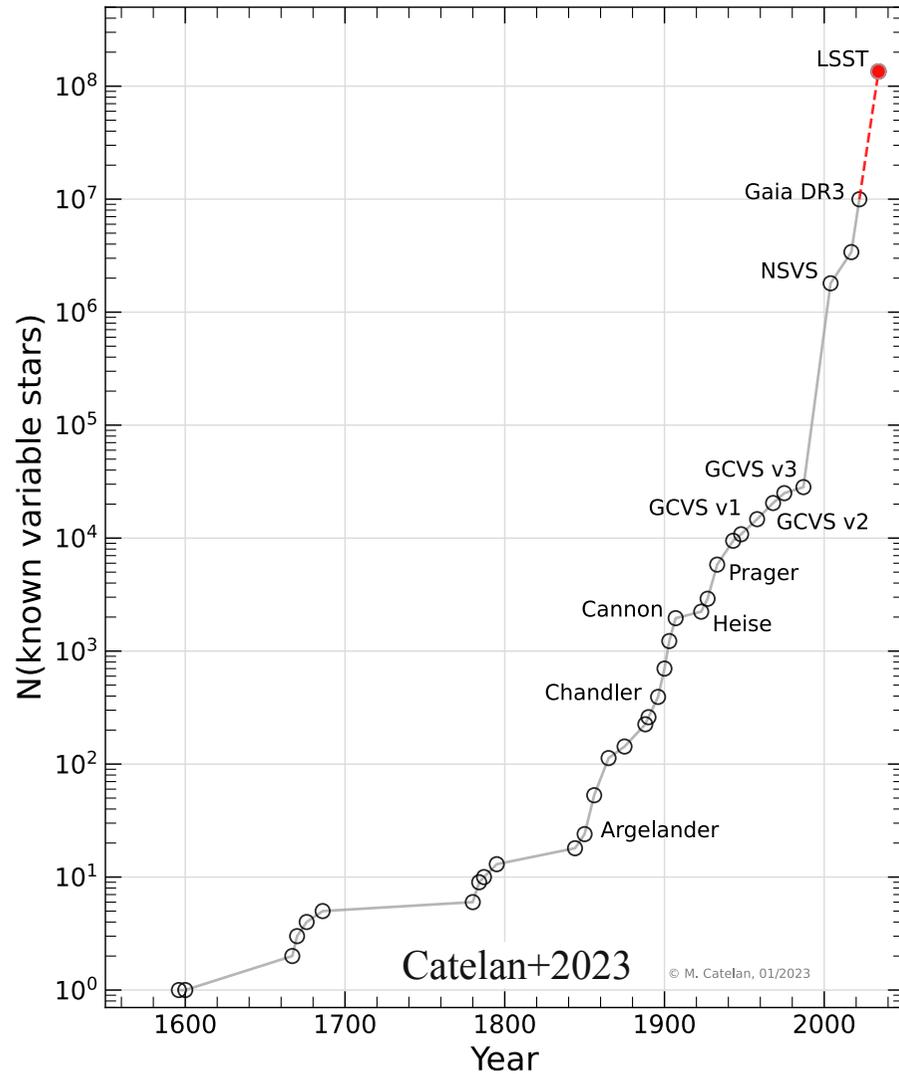


Gaia DR3

Eyer+2022

Variable star numbers

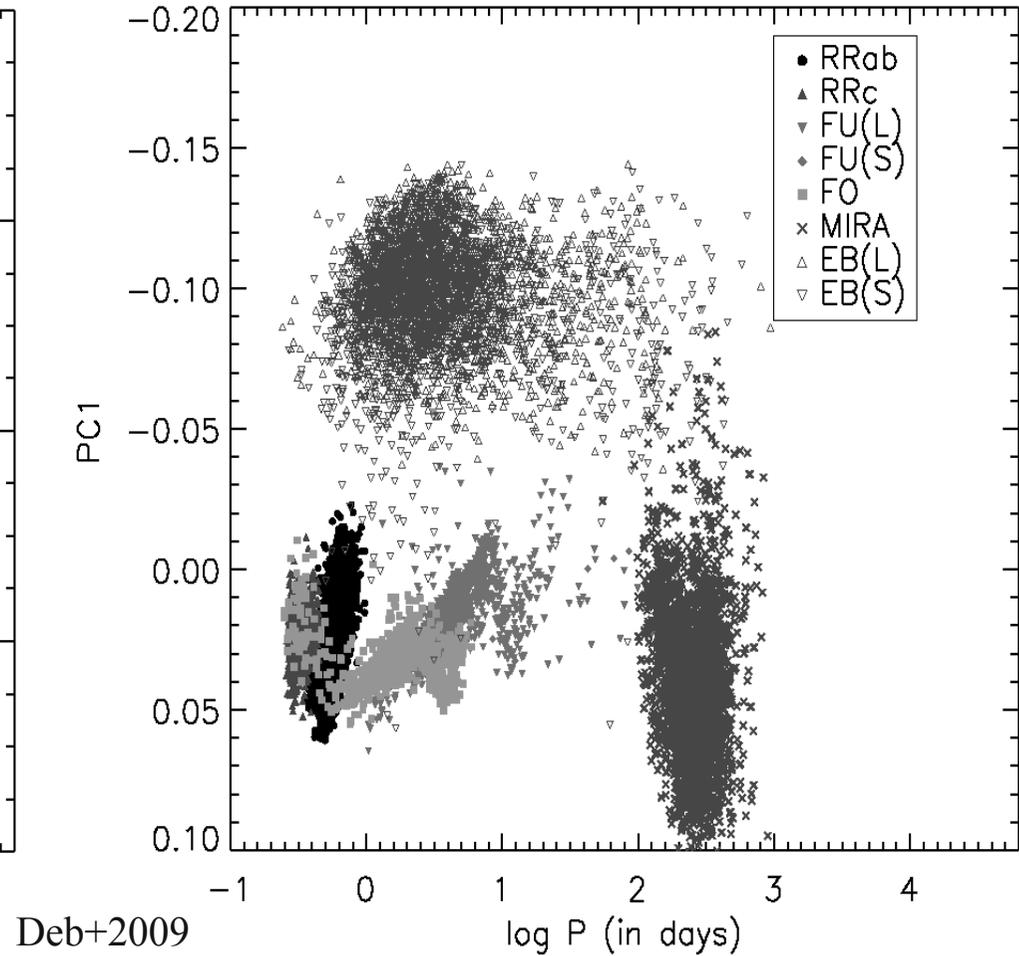
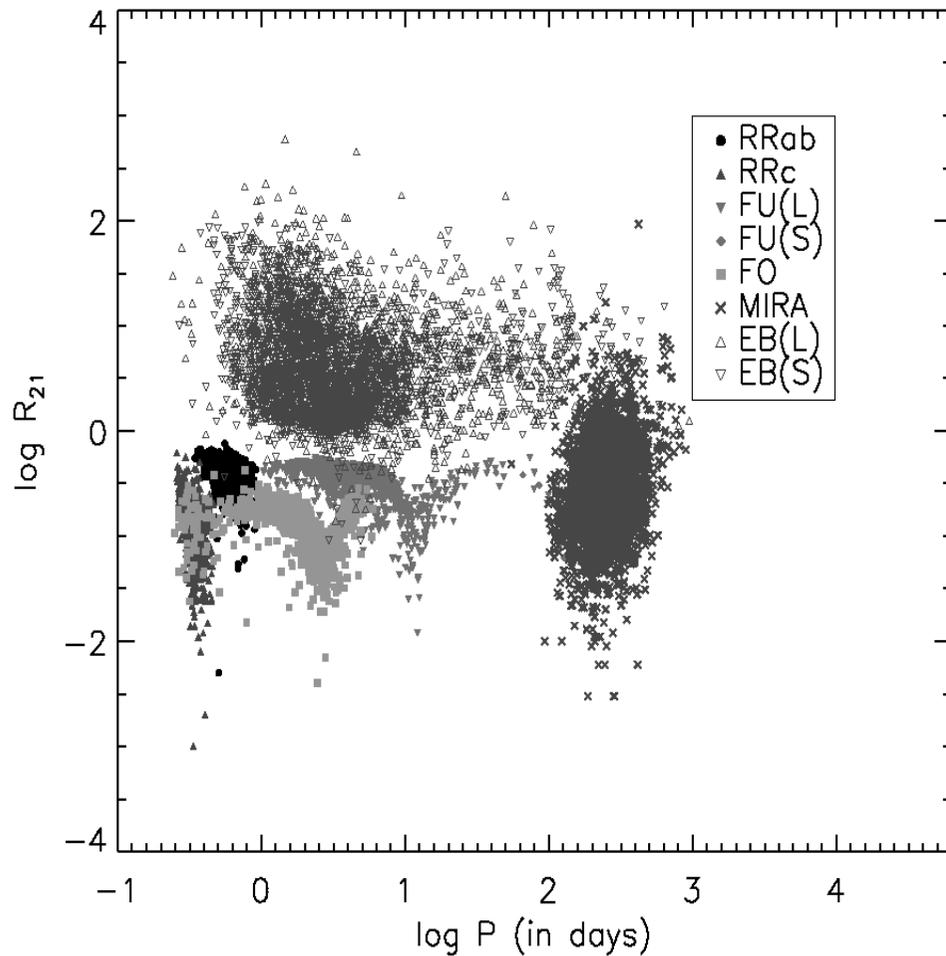
Now and with LSST...



Variable star classification



Traditional Fourier/PCA analysis for periodic variable stars

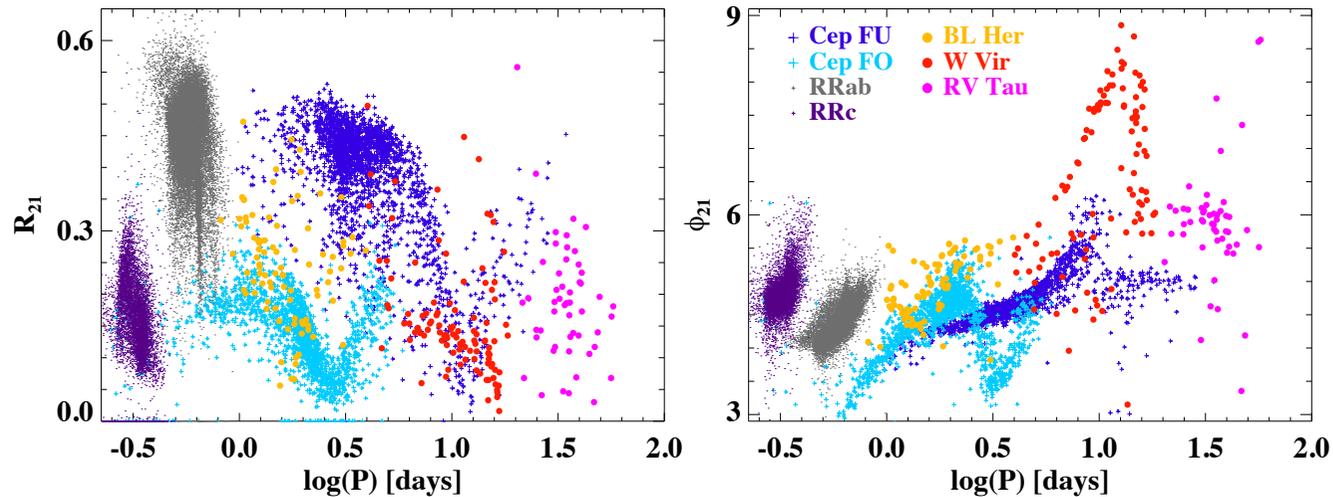
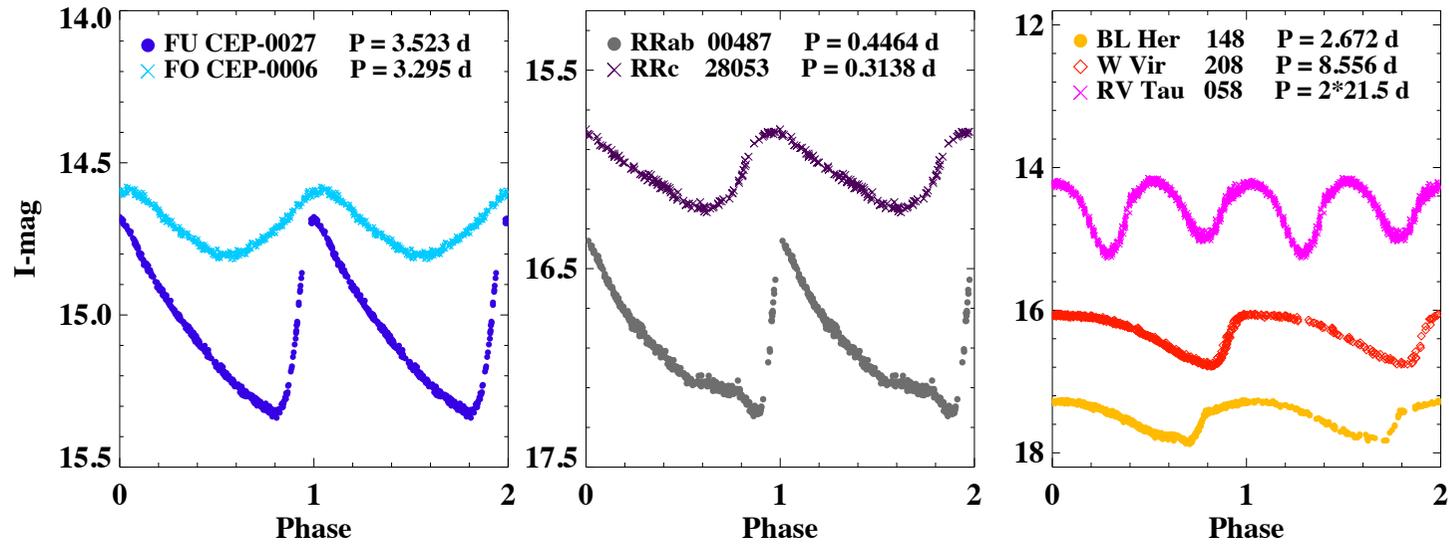


Variable star classification

Year	Catalog / Survey	Title
2013	Gaia Mission (Preliminary Data)	<i>Gaia: The astrometric and photometric mission</i>
2016	OGLE-IV	<i>The OGLE-IV Catalog of Galactic and Magellanic Cloud Variables</i>
2017	GCVS 5th Edition	<i>General Catalogue of Variable Stars, 5th Edition</i>
	CRTS	<i>The Catalina Real-Time Transient Survey: Variable Stars in the Catalina Sky Survey</i>
2018	Gaia DR2	<i>Gaia Data Release 2: The astrometric solution</i>
	Pan-STARRS1	<i>The Pan-STARRS1 Transients and Variable Stars</i>
	VVV	<i>VVV Variable Star Catalog</i>
	VMC	<i>The VISTA Magellanic Clouds Survey: Variable Stars and Their Evolutionary Pathways in the Magellanic Clouds</i>
2019	ZTF	<i>The Zwicky Transient Facility Bright Transient and Variable Star Catalog</i>
	ASAS-SN	<i>The ASAS-SN Catalog of Variable Stars</i>
2020	TESS	<i>TESS Variable Star Catalog: Early Results from the First Year of Data</i>
2022	Gaia DR3	<i>Gaia Data Release 3</i>
2025 (expected)	LSST	<i>Variable Stars in the LSST Era</i>

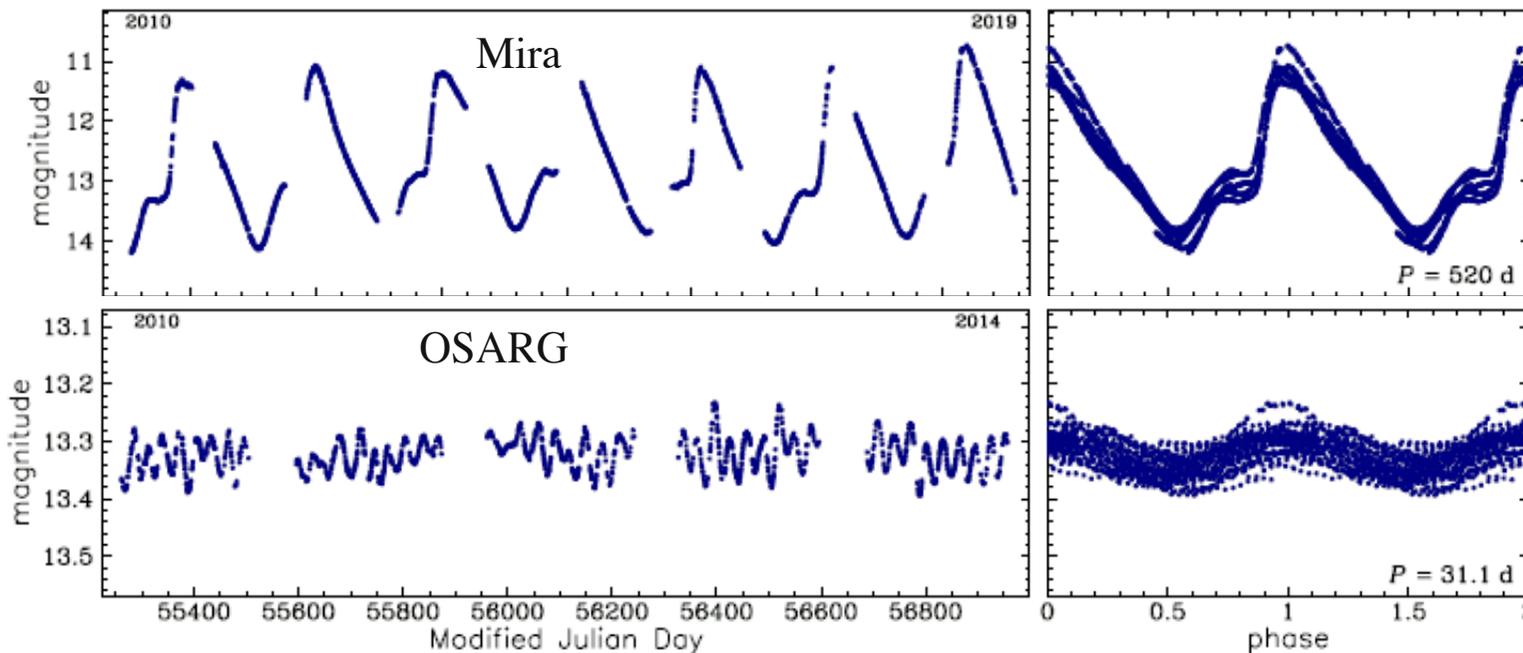
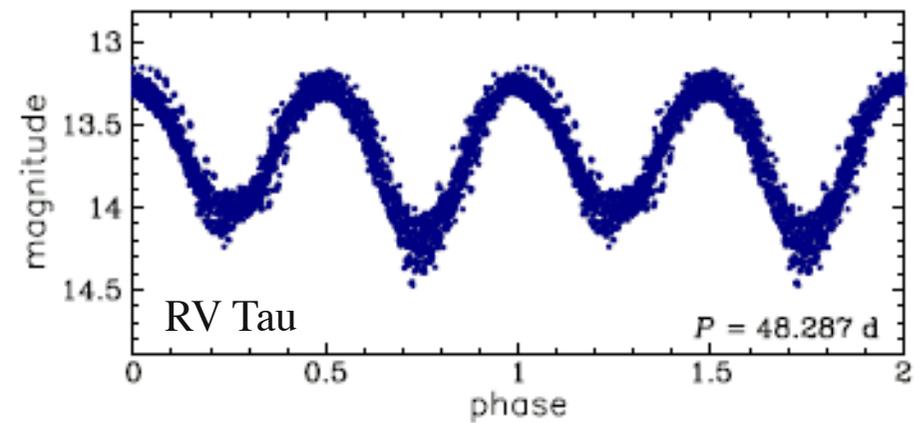
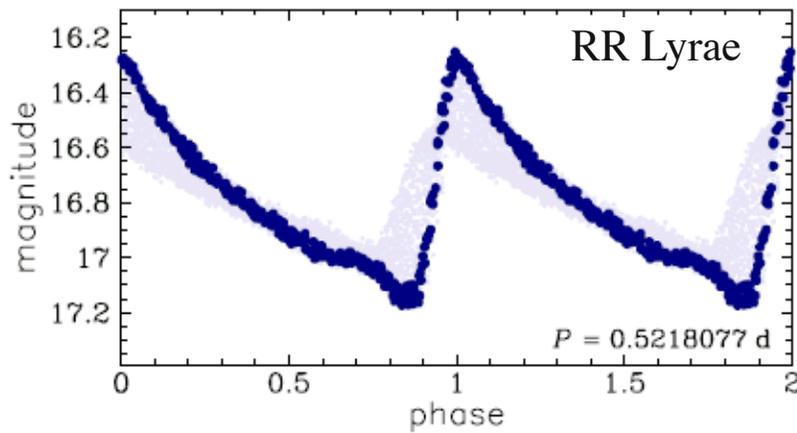
Variable star classification

Light curves of periodic variables



Variable star classification

Light curves of periodic variables



Credit:
OGLE

Variable star classification

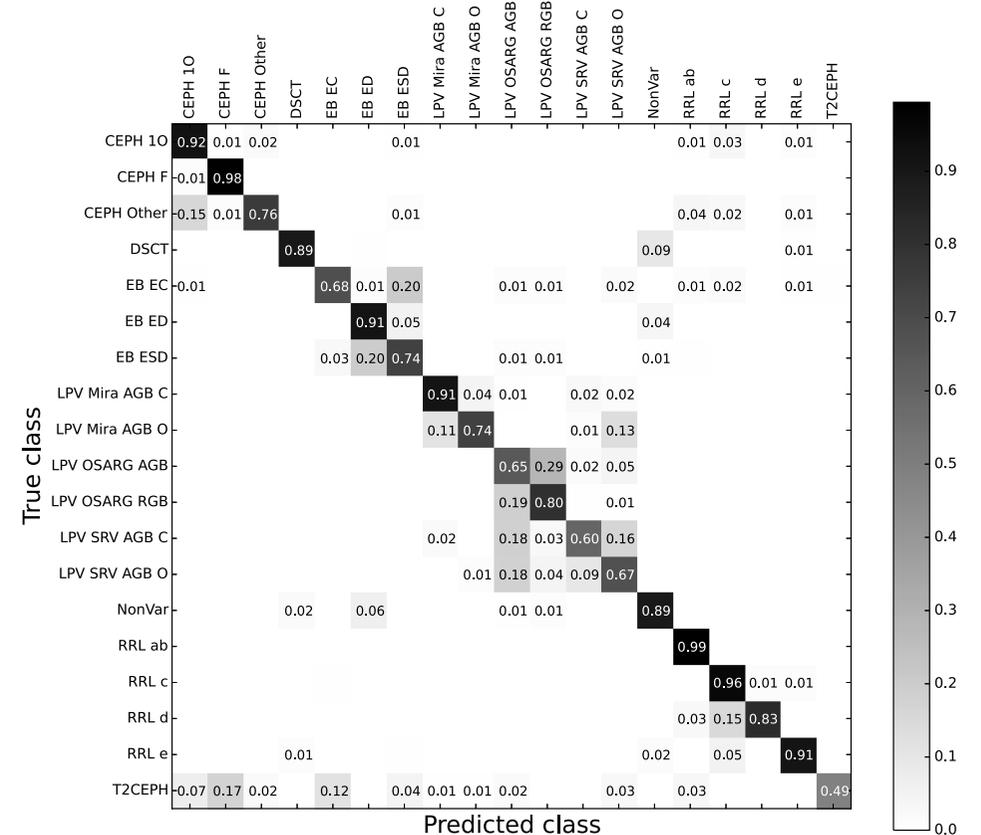
Random-Forest classifier — Light curve features

Feature	Description	Estimated by LC
Period	Photometric period determined by the Lomb-Scargle periodogram	Y
Parallax	Photometric parallax provided by <i>Gaia</i> DR3	
Error	Parallax uncertainty provided by <i>Gaia</i> DR3	
$(B_P - R_P)_0$	$(G_{BP} - G_{RP})_0$, dereddened color determined based on <i>Gaia</i> DR3	
M_{WG}	<i>G</i> -band (330 nm to 1050 nm) absolute Wesenheit magnitudes determined based on <i>Gaia</i> DR3	
Amp.	Normalized peak-to-peak amplitude determined from the eighth-order Fourier fitting of the LCs	Y
γ_2	Kurtosis = $\frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2)^2} - 3$	Y
γ_1	Skewness = $\frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2)^{3/2}}$	Y
Q_{31}	The normalized difference between 3rd quartile (75%) and 1st quartile (25%) in LCs	Y
W	Shapiro-Wilk normality test statistics	Y
K	Stetson K index, calculated using LCs	Y
Std	The normalized weighted standard deviation of the LCs	Y
R_{21}	a_2/a_1 , 2nd to 1st amplitude ratio obtained from the eighth-order Fourier fitting	Y
R_{31}	a_3/a_1 , 3rd to 1st amplitude ratio obtained from the eighth-order Fourier fitting	Y
ϕ_{21}	$\phi_2 - 2\phi_1$, the phase difference between 2nd to 1st phase obtained from the eighth-order Fourier fitting	Y
ϕ_{31}	$\phi_3 - 3\phi_1$, the phase difference between 3rd to 1st phase obtained from the eighth-order Fourier fitting	Y
$W_1 - W_3$	$W_1 - W_3$ color determined based on <i>WISE</i>	
$W_1 - W_4$	$W_1 - W_4$ color determined based on <i>WISE</i>	
M_{W_1}	The absolute W_1 -band Wesenheit magnitude	

TESS (Gao+2024)

ALeRCE (Sánchez-Sáez+2021)

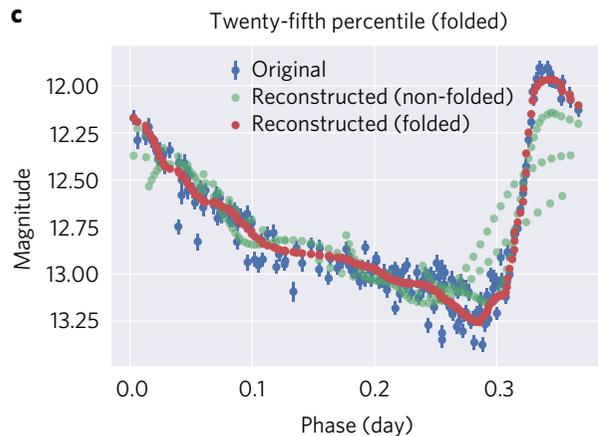
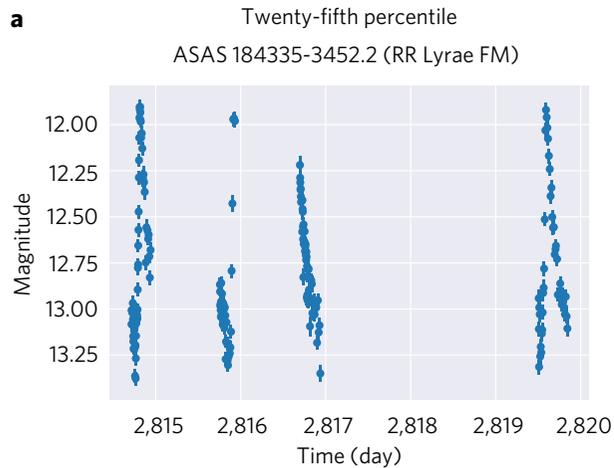
OGLE (Kim & Bailer-Jones+2016)



Variable star classification

Recurrent Neural Networks

Encoder-decoder architecture (Naul+2018)



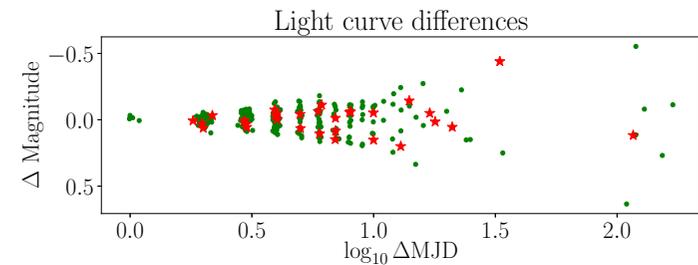
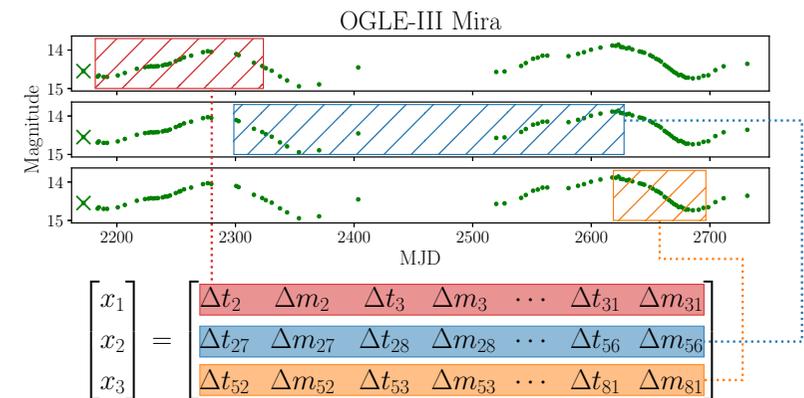
a

	ASAS				
Beta Persei	136	0	0	0	1
Classical Cepheid	5	43	2	3	0
RR Lyrae FM	2	1	244	0	1
Semiregular PV	1	1	4	69	1
W Ursae Major	2	0	0	0	645

True label

Predicted label

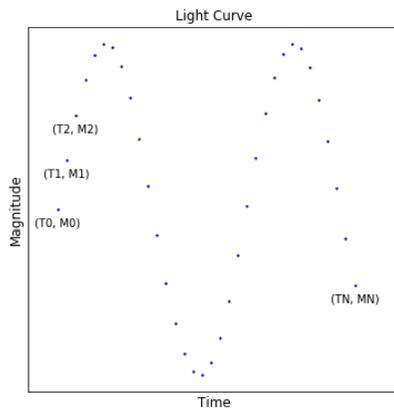
dmdt matrix — OGLE (Becker+2020)
CRTS (Kang+2023)



Variable star classification

Convolutional neural networks (CNN)

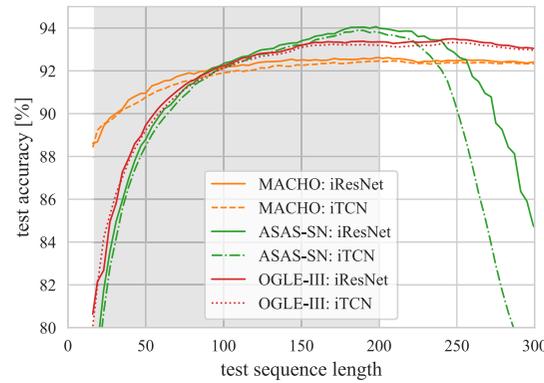
dmdt matrix based inputs (Aguirre+2019)



$$\begin{bmatrix} T_0, T_1, \dots, T_N \\ M_0, M_1, \dots, M_N \end{bmatrix}$$

$$\begin{bmatrix} 0, (T_1 - T_0), \dots, (T_N - T_{N-1}) \\ 0, (M_1 - M_0), \dots, (M_N - M_{N-1}) \end{bmatrix}$$

$$\begin{bmatrix} T'_0, T'_1, \dots, T'_{N'} \\ M'_0, M'_1, \dots, M'_{N'} \end{bmatrix}$$



Zhang+2021

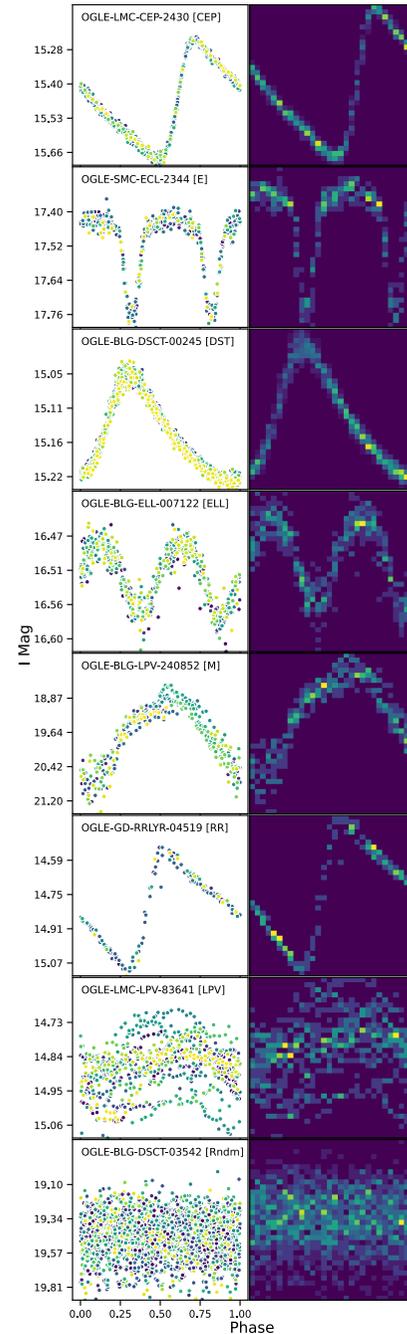
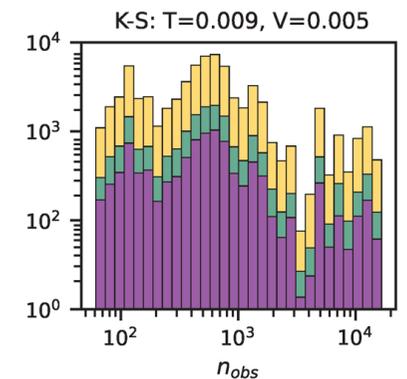


Image-based CNNs
Of OGLE light curves
Szklenár+2021,
Monsalves+2024

Dmdt-images
CRTS light curves
Mahabal+2017

Train Test Validation



Monsalves+2024

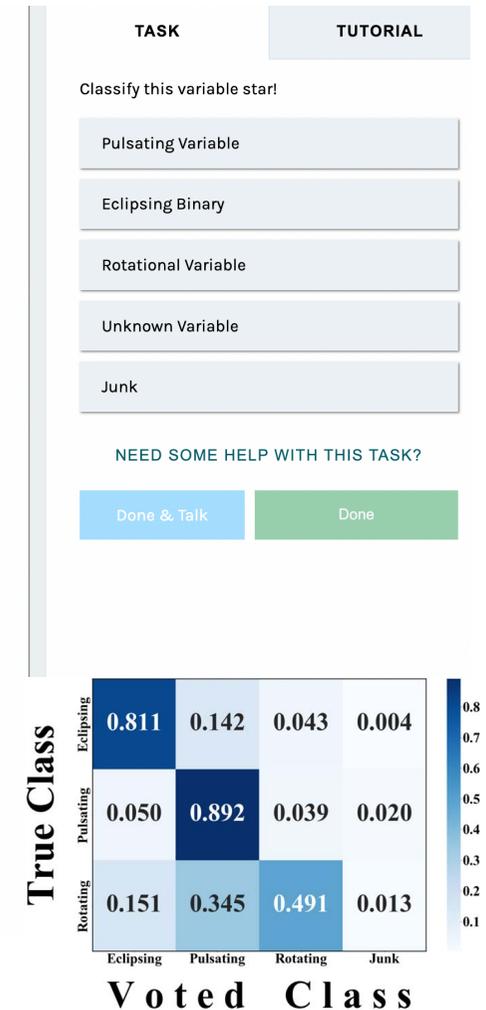
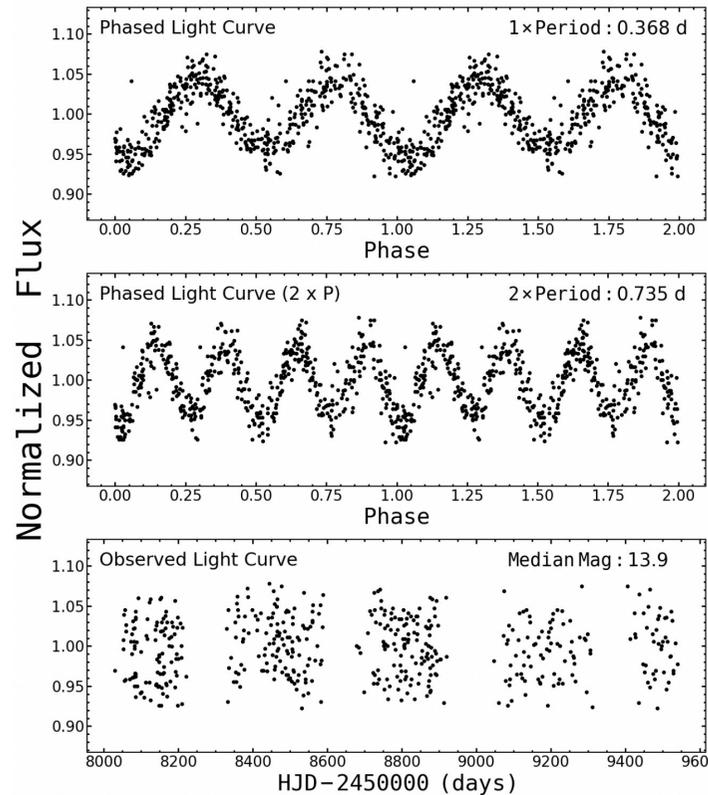
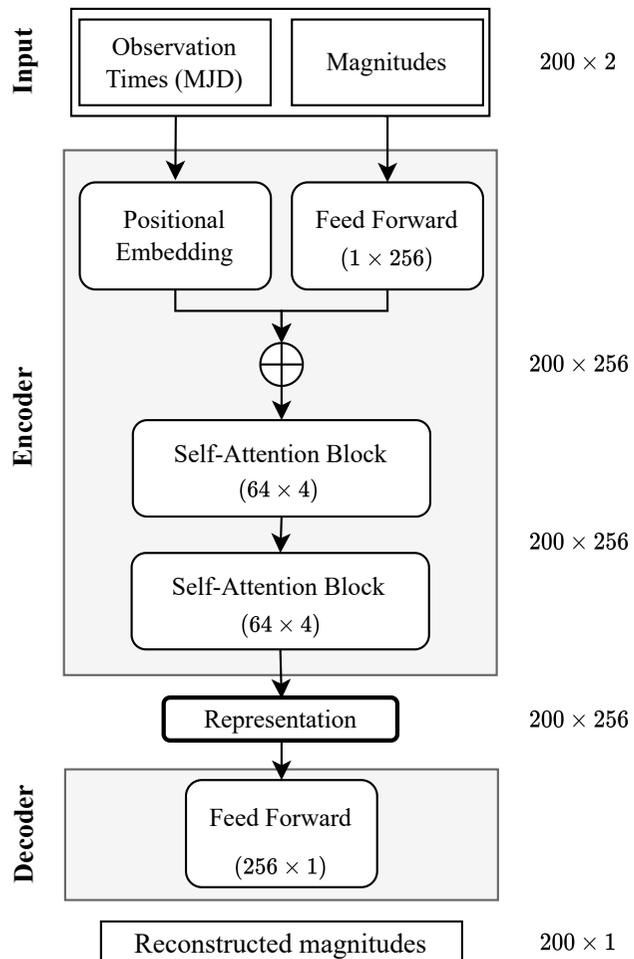
Variable star classification

Transformer-based classifier

Citizen ASAS-SN data release

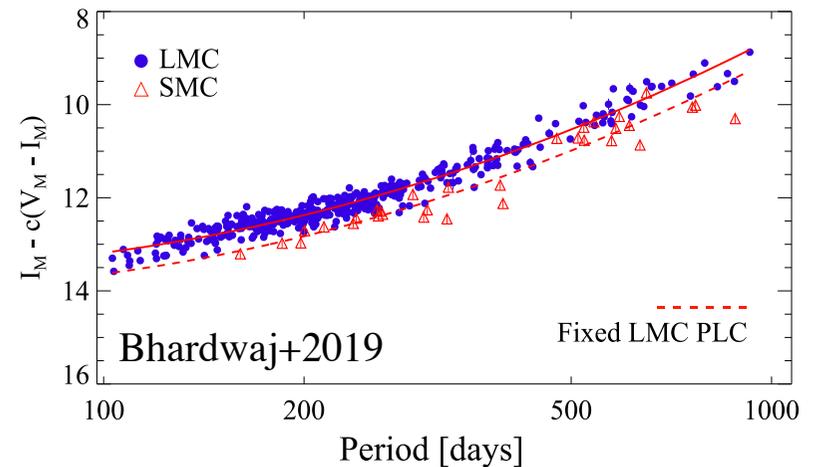
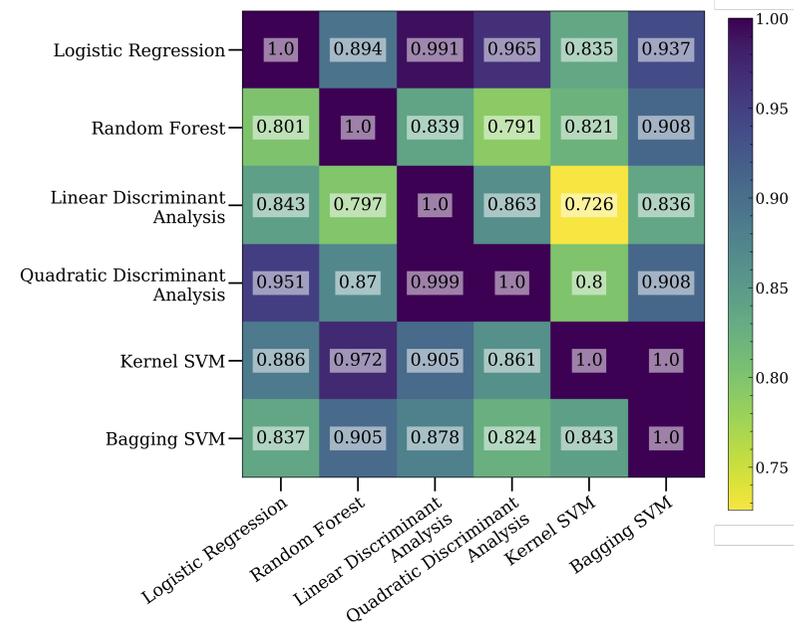
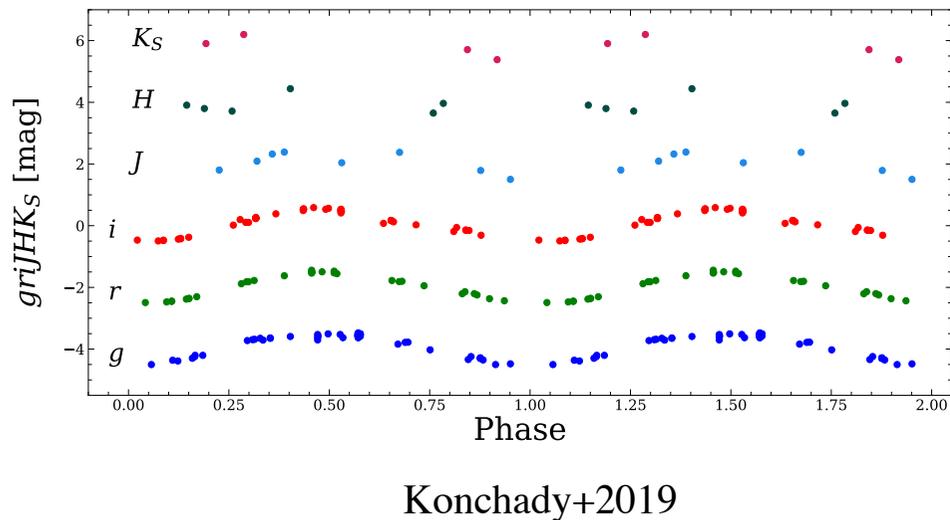
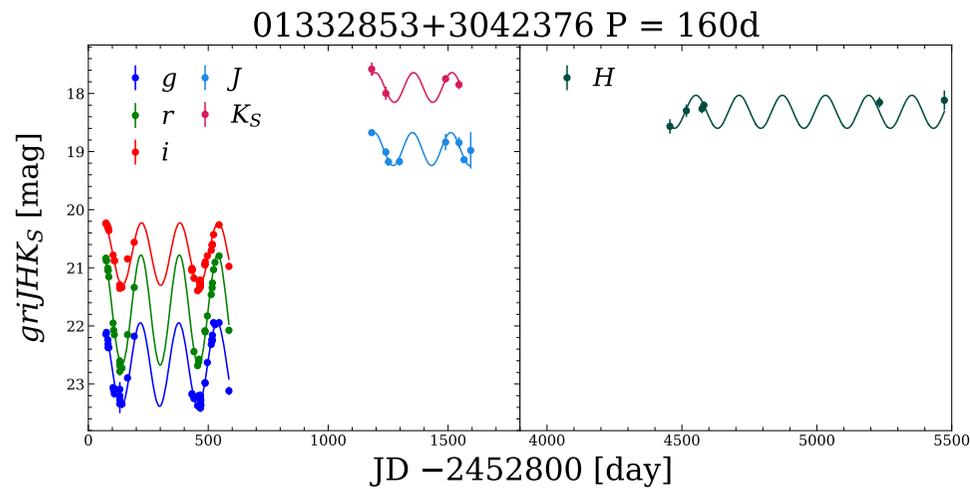
(Christy+2022)

ASTROMER — (Donoso-Oliva+2023)



Variable star classification

Specific object studies — Miras



Fundamental parameters in an instant...

Cepheid mass discrepancy problem

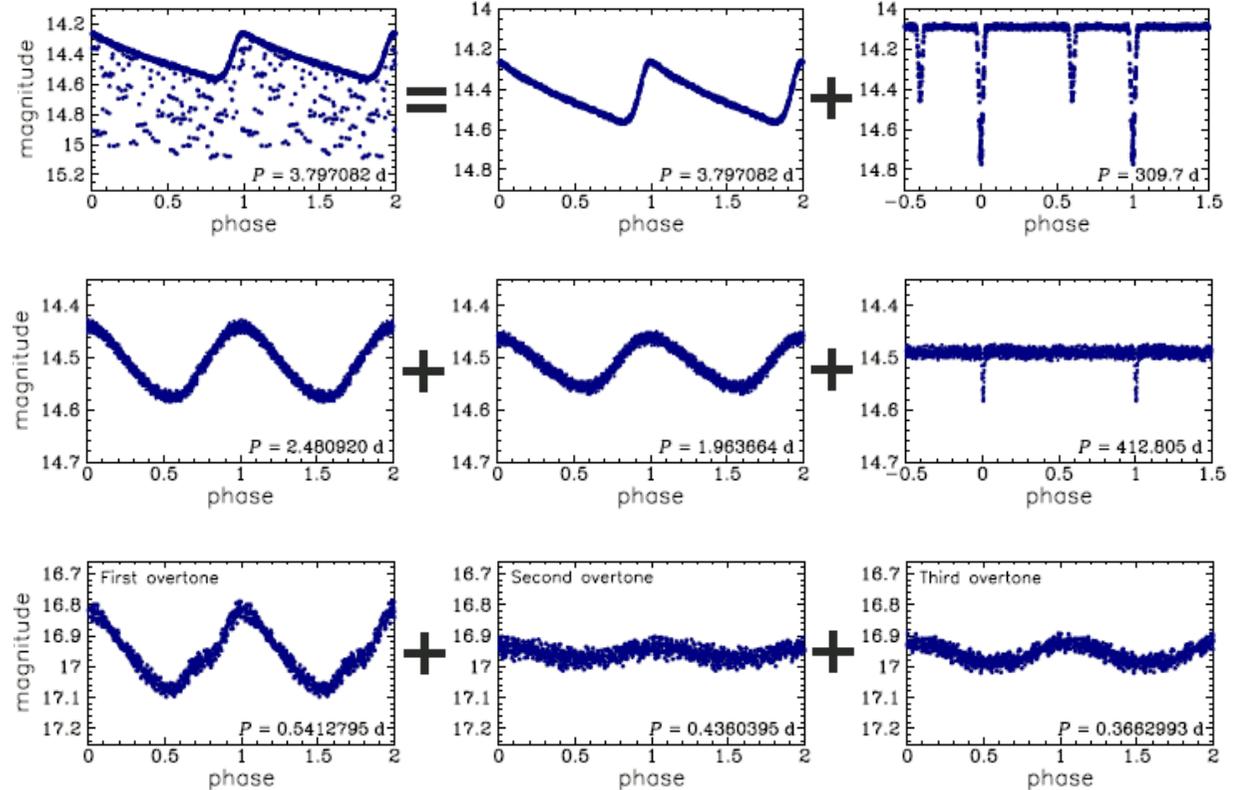
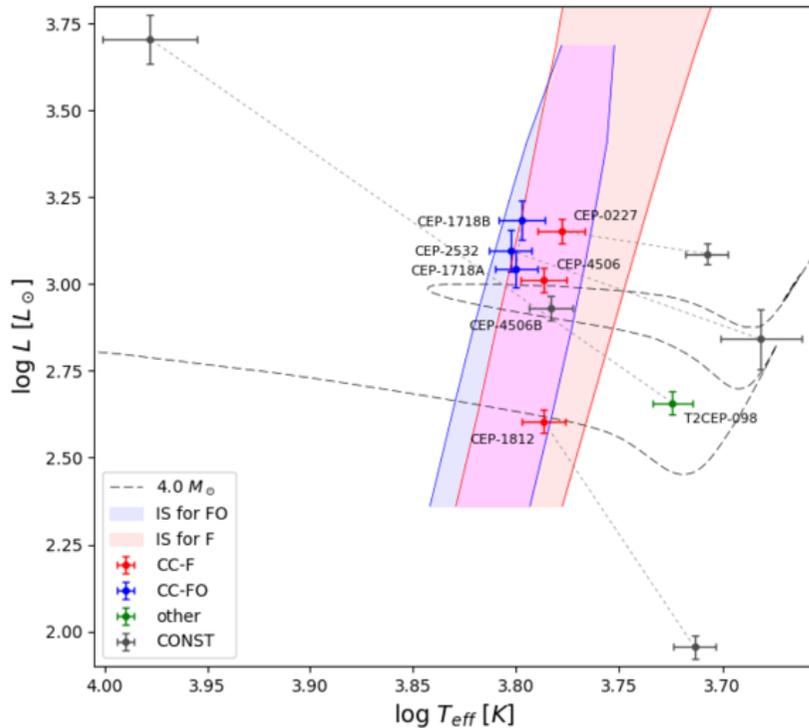
Mass-Luminosity relations

Period-mass-radius relations

Dynamical mass of a classical Cepheid to an unprecedented 1% accuracy OGLE-LMC-CEP-0227

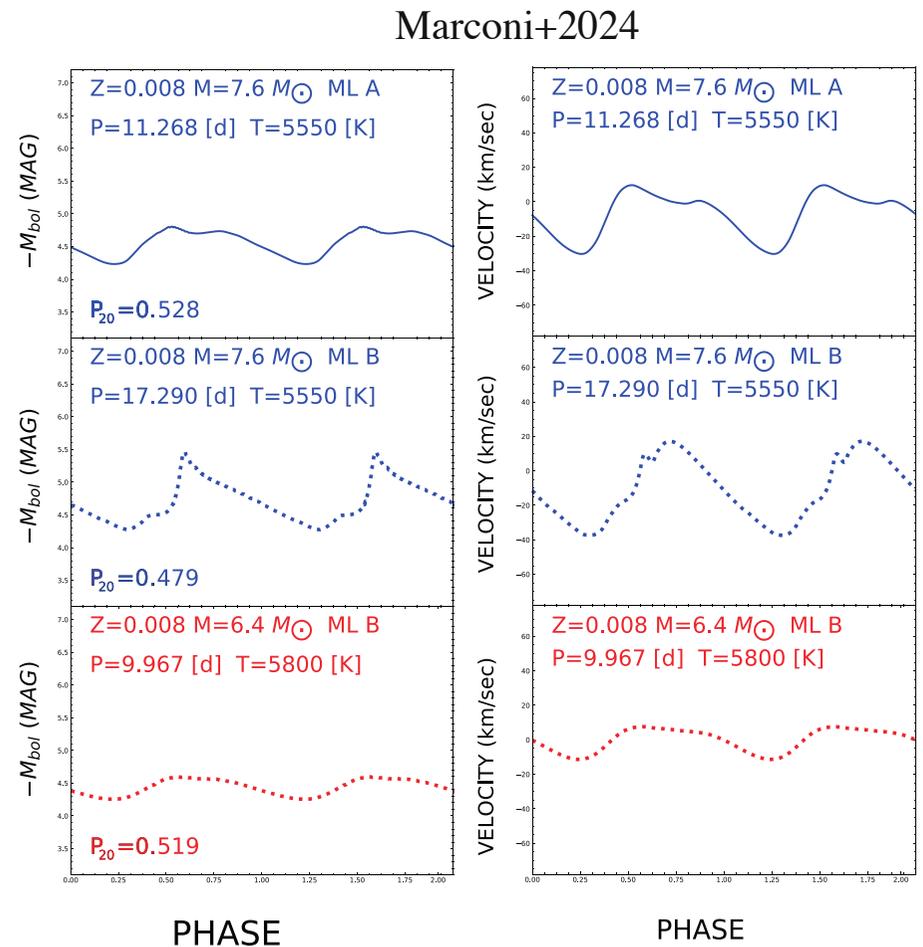
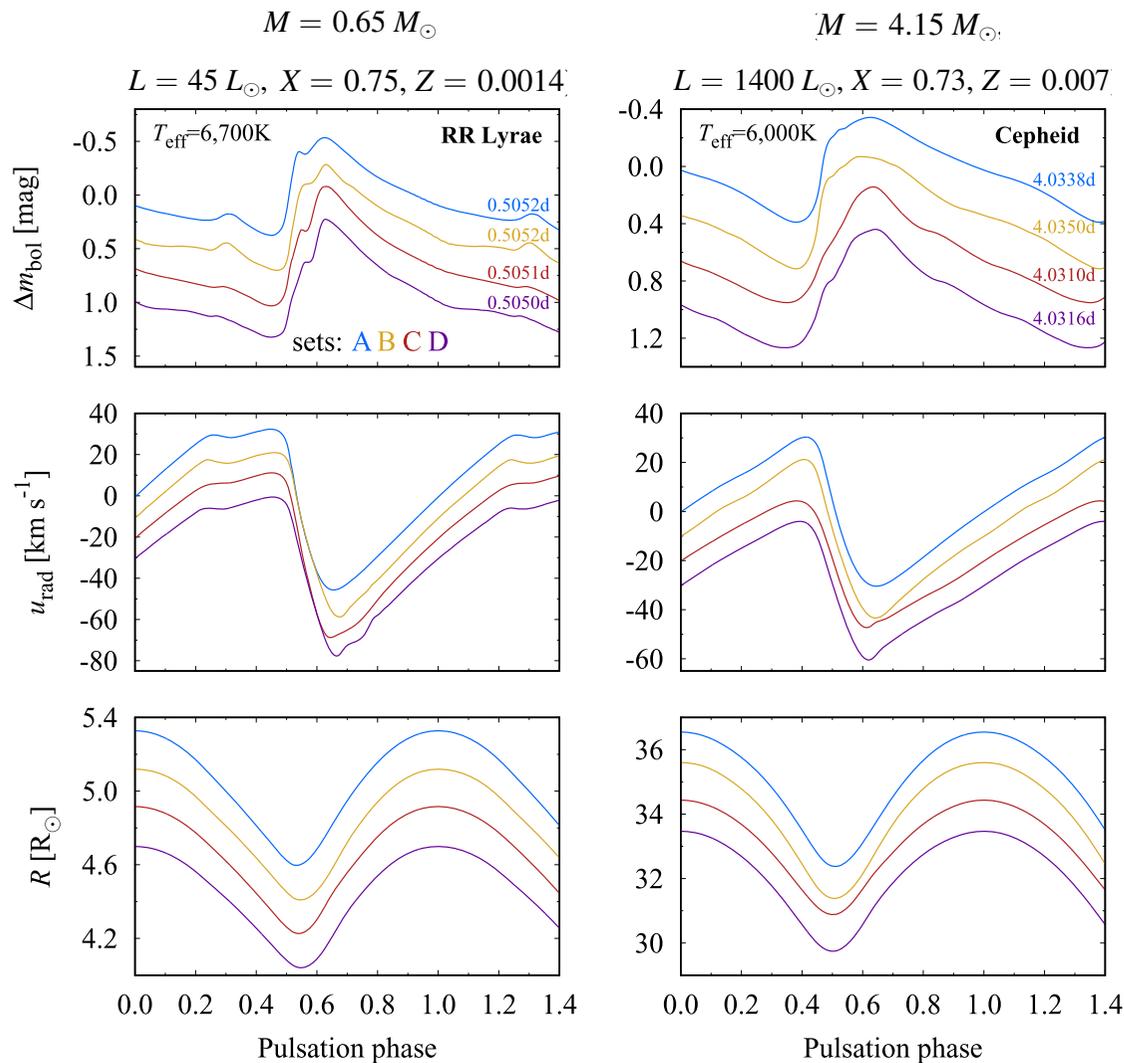
(Pietrzyński et al. 2010, Nature)

Binary Cepheids (Pilecki+2019)



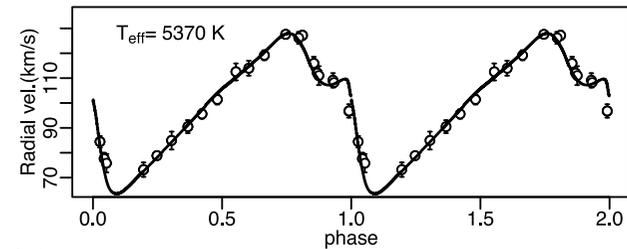
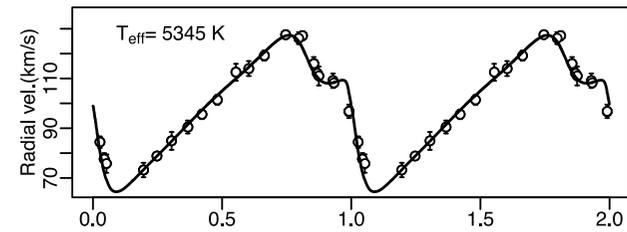
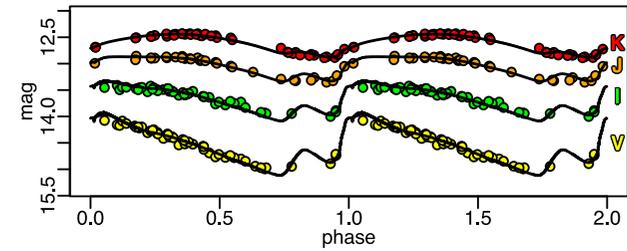
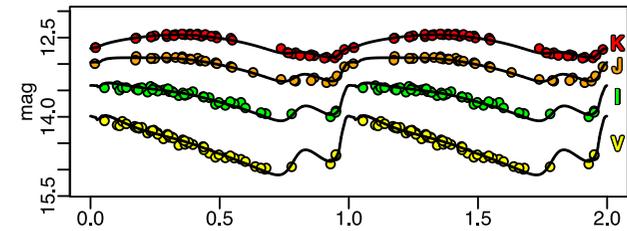
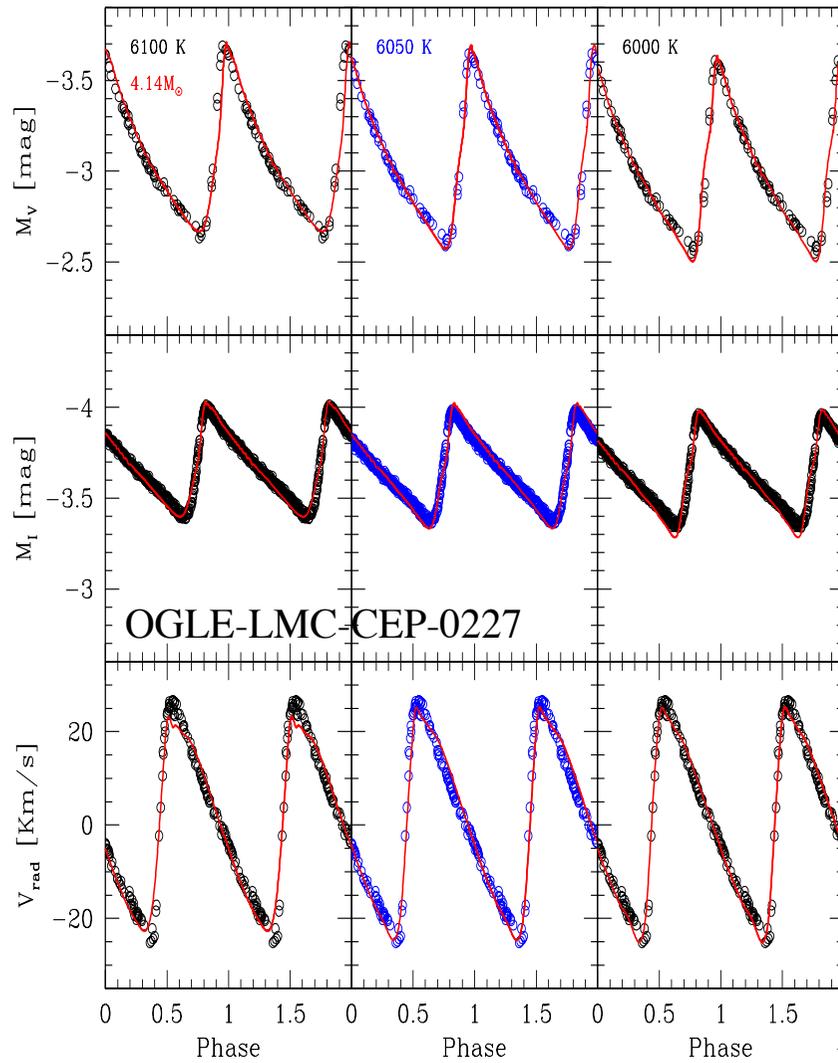
Predicted fundamental parameters

Non-linear time-dependent hydrodynamical models can reproduce light and radial velocity variations of Cepheid and RR Lyrae variables (Bono+2000, Marconi+2015, Paxton+2019)



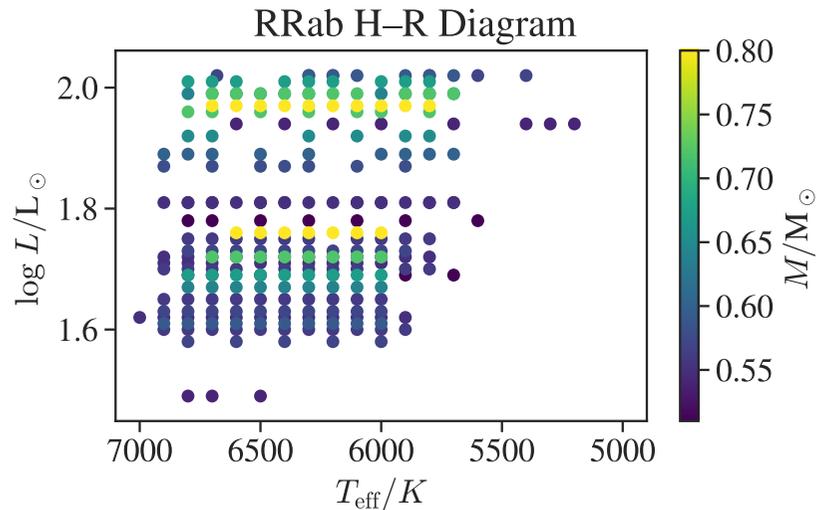
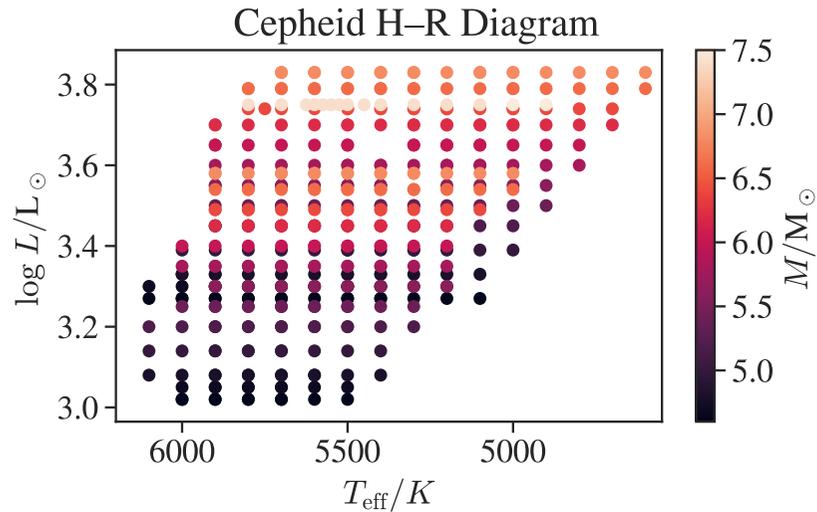
Predicted fundamental parameters

Model fitting approach



Predicted fundamental parameters

Model fitting approach to ensemble based analyses



Light curve structure parameters

$$M(t) = A_0 + \sum_{k=1}^N A_k \cos(k\omega t + \phi_k)$$

I - and V -band amplitudes, acutenesses, skewnesses, and the coefficients A_1 , A_2 , and A_3 .

$$f(M, L, R, T_{\text{eff}}, \dots) = [\text{period, light curve structure}]$$

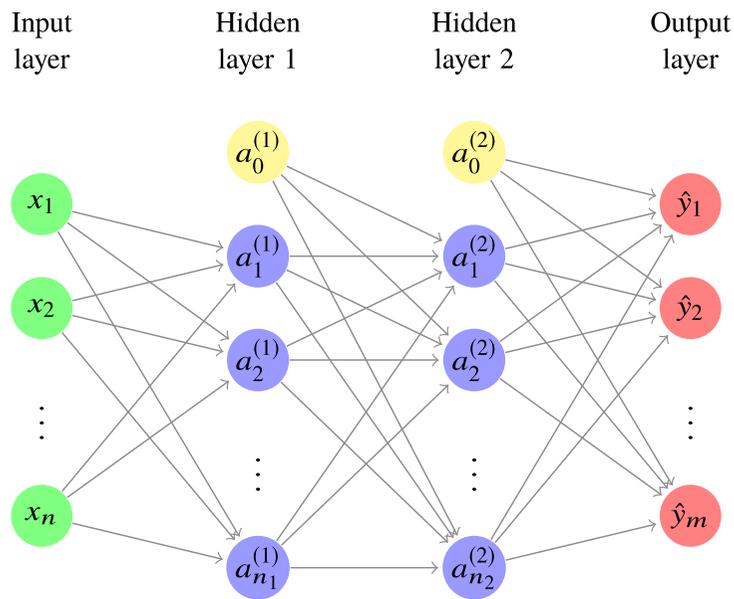
$$g = f^{-1}$$

$$g(\text{period, light curve structure}) = [M, L, R, T_{\text{eff}}, \dots]$$

Predicted fundamental parameters

Ensemble based analyses

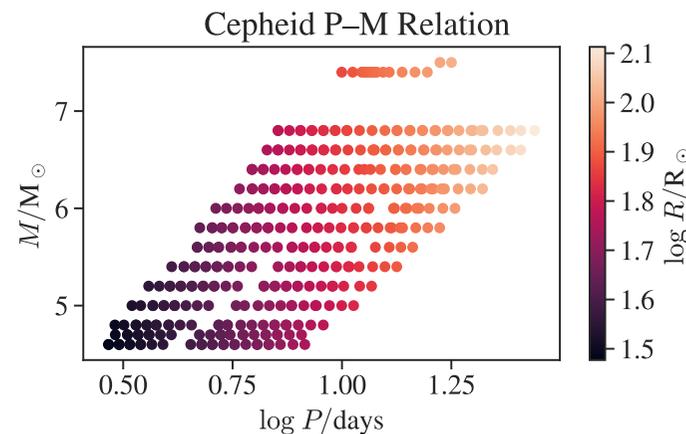
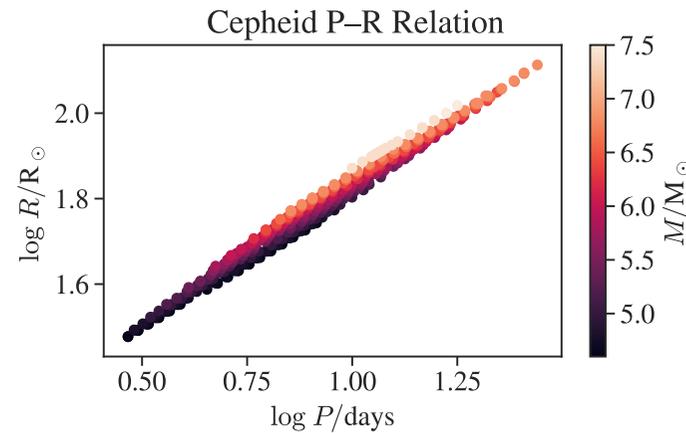
ANN trained on the theoretical models



Two-fold cross-validation

Bellinger, Kanbur, Bhardwaj+2020

Model assessment



Linear model

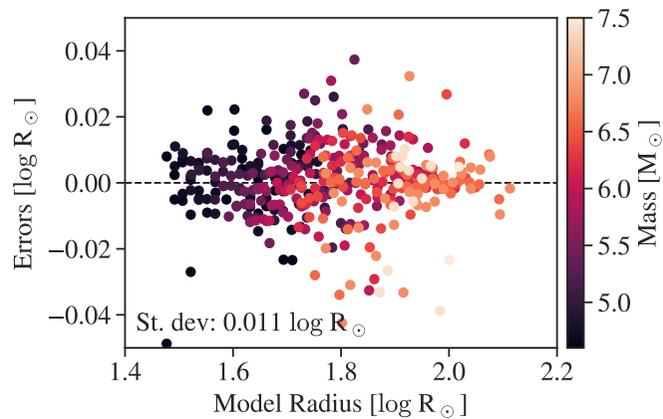
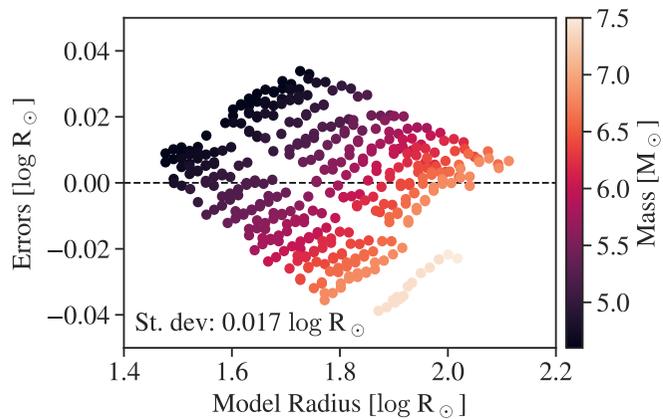
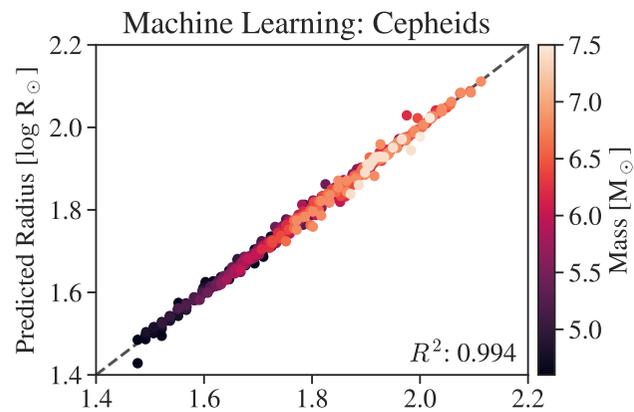
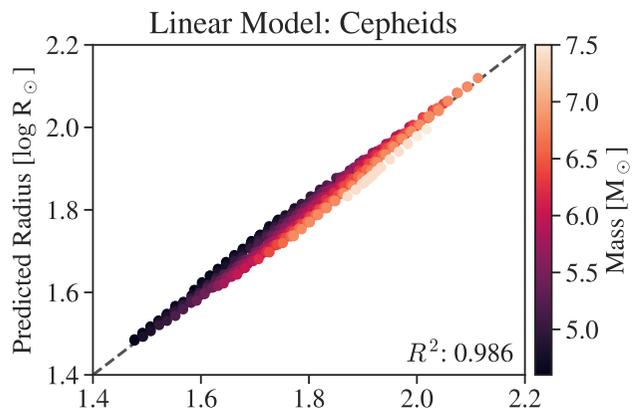
$$y = a + b \log P$$

$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2}$$

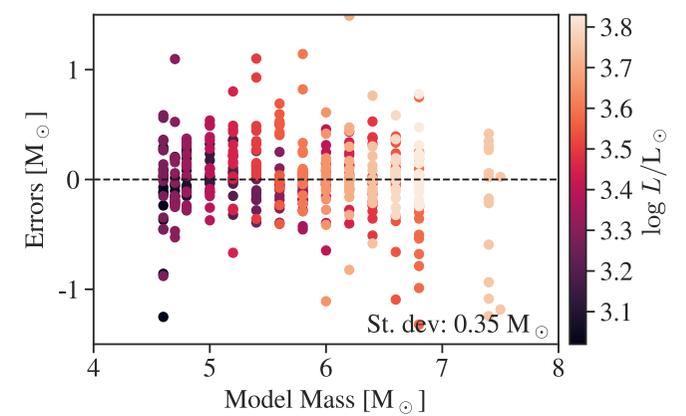
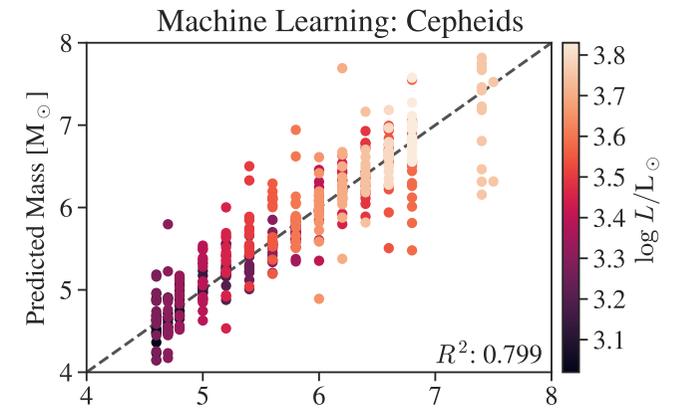
Predicted fundamental parameters

Ensemble based analyses

Radius



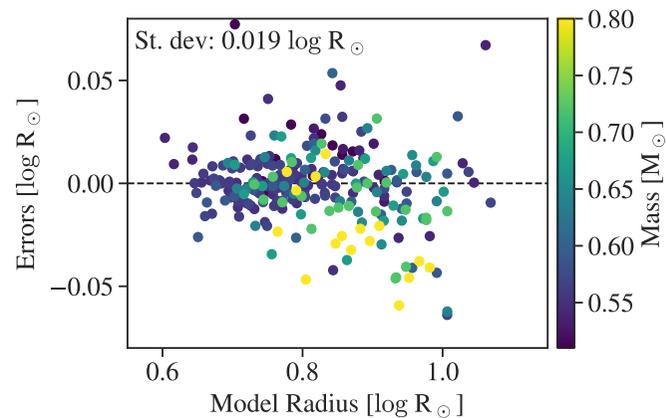
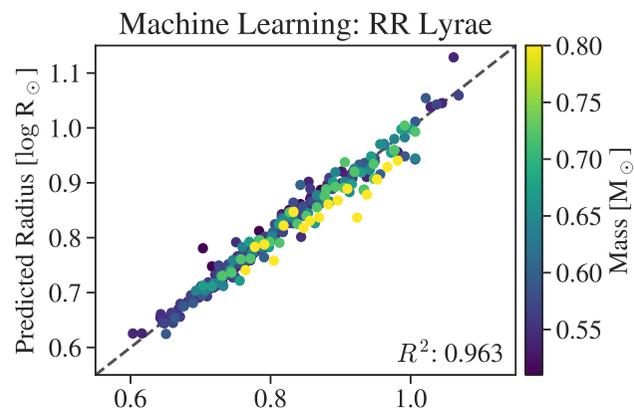
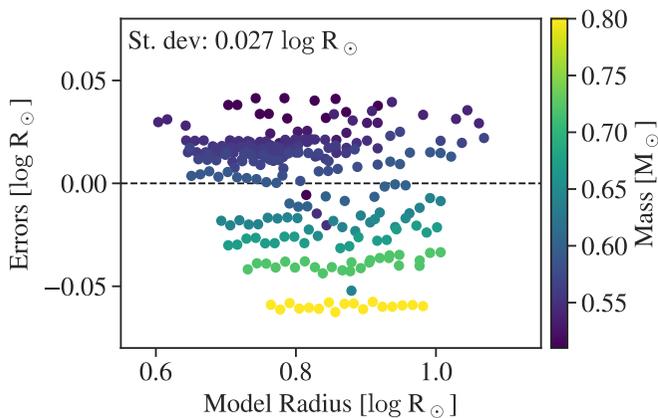
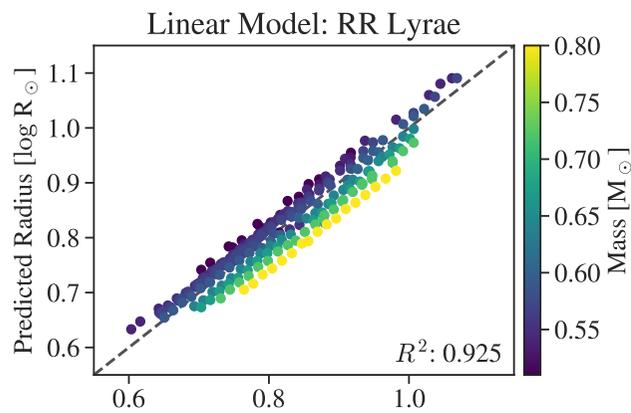
Mass



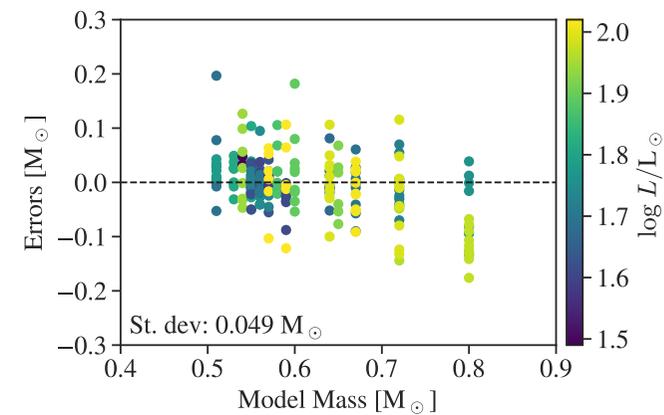
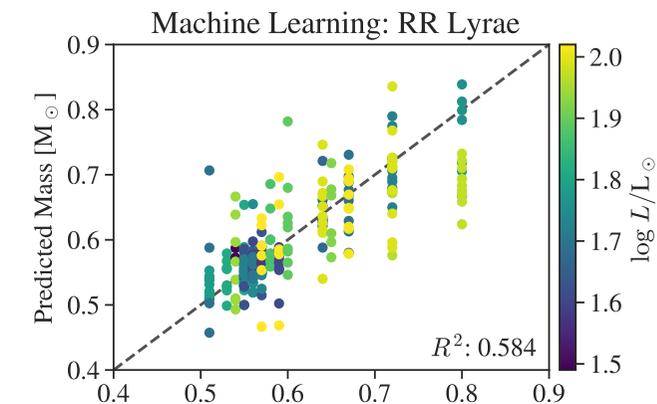
Predicted fundamental parameters

Ensemble based analyses

Radius



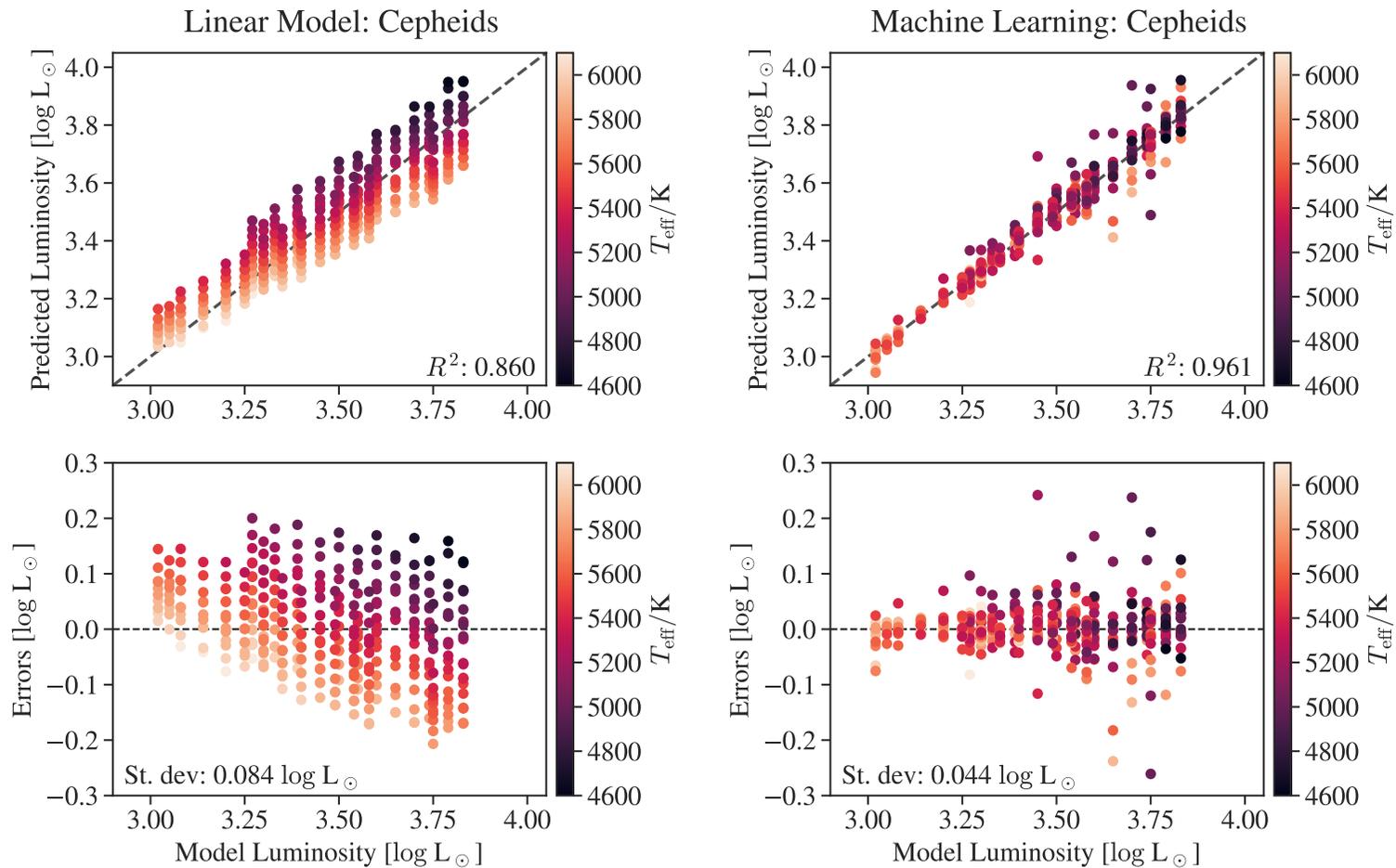
Mass



Predicted fundamental parameters

Ensemble based analyses

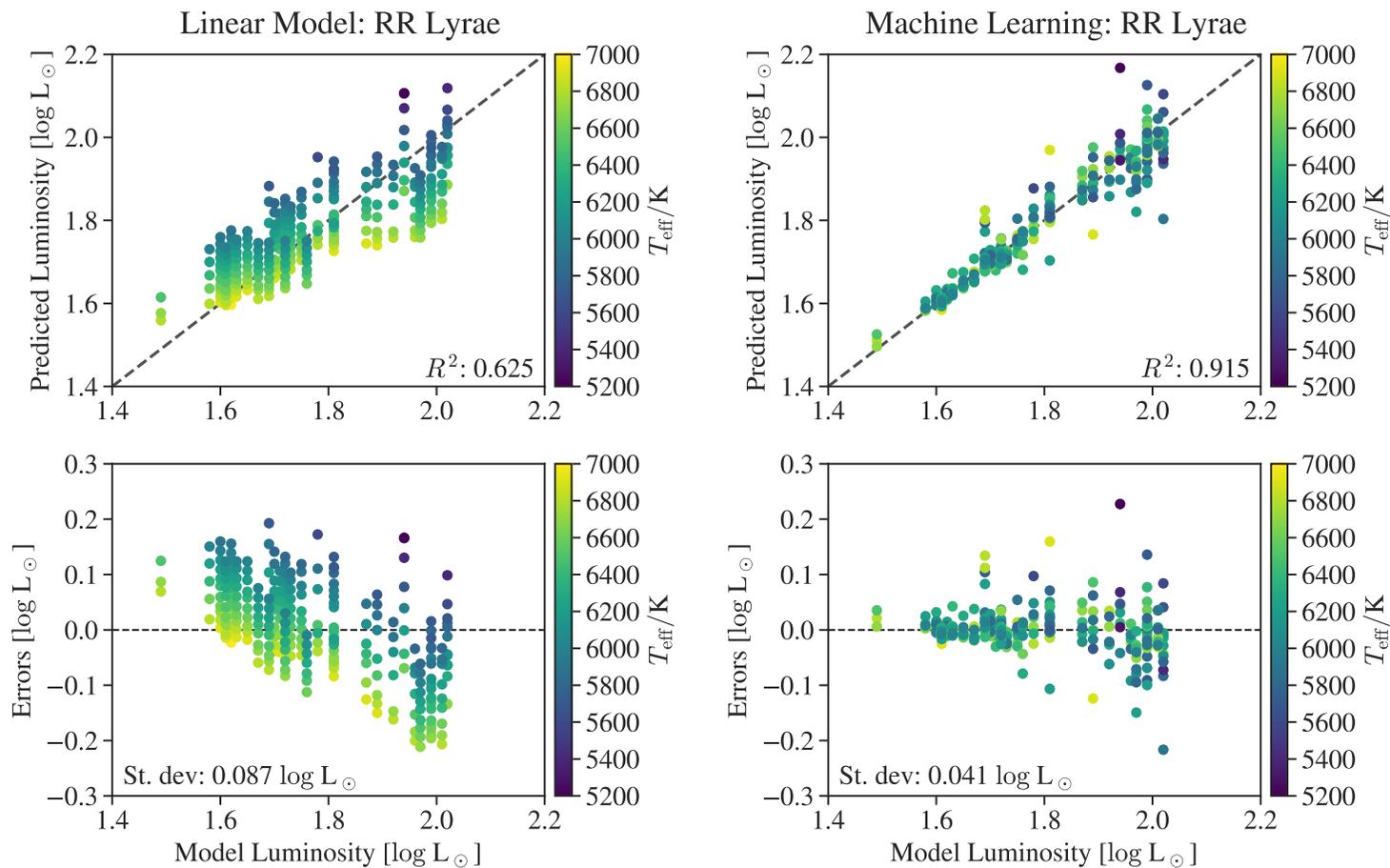
Predicted luminosities of Cepheid and RR Lyrae stars



Predicted fundamental parameters

Ensemble based analyses

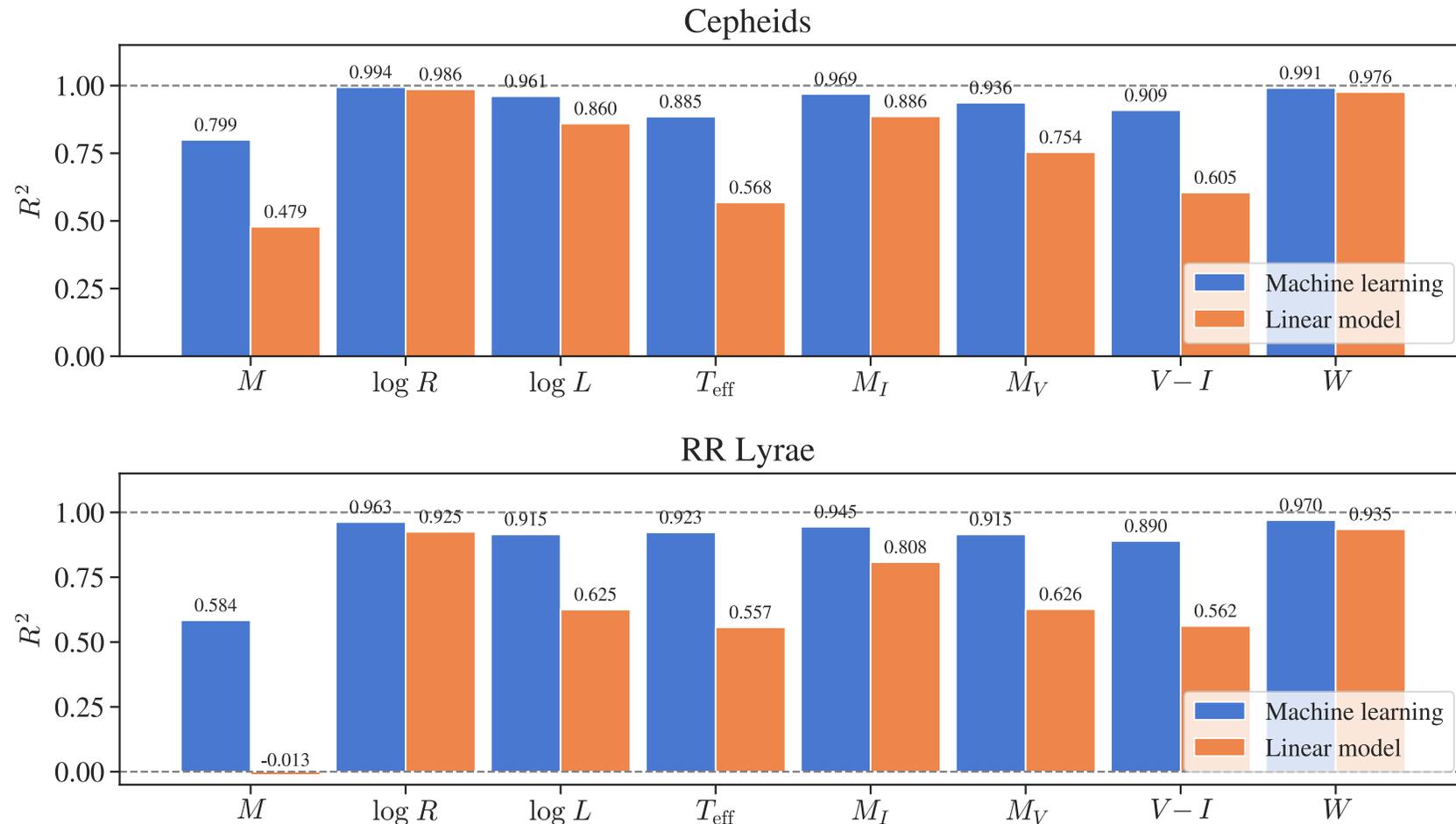
Predicted luminosities of Cepheid and RR Lyrae stars



Predicted fundamental parameters

Ensemble based analyses

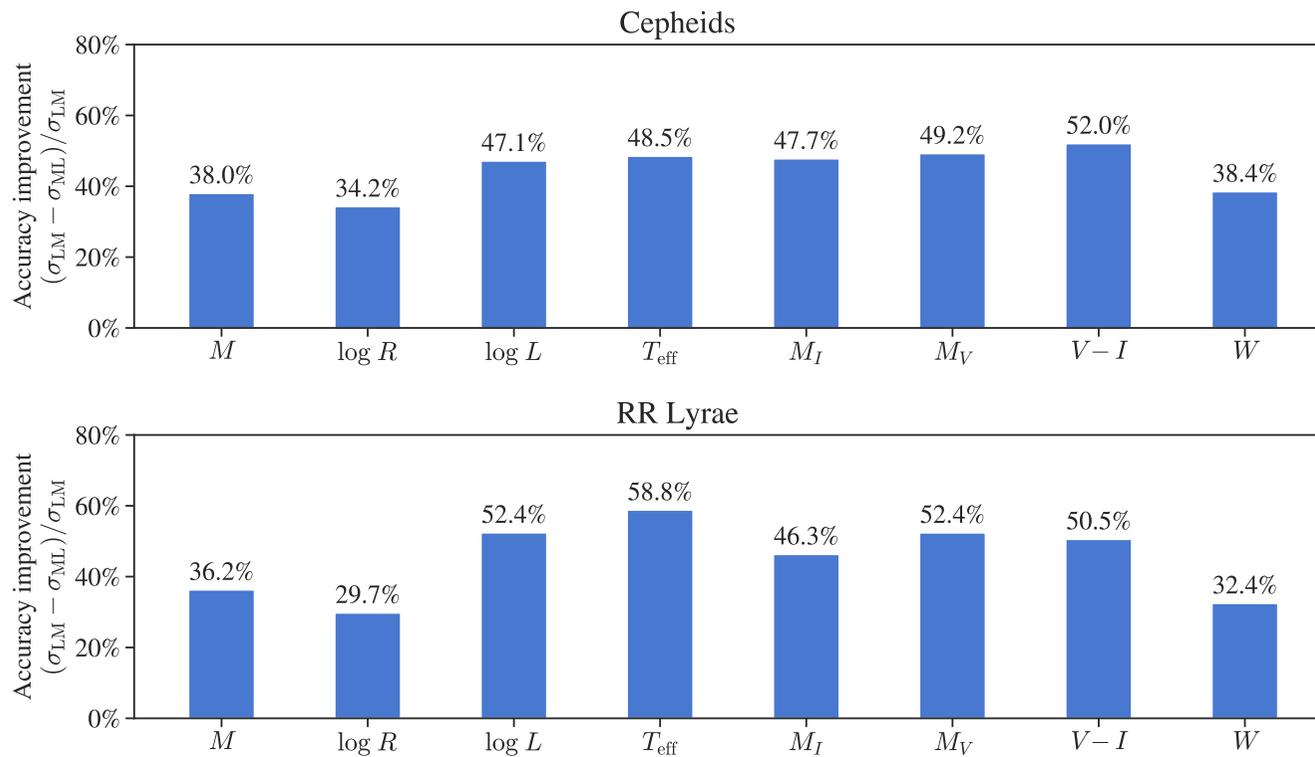
Light Curve Structure Importances



Predicted fundamental parameters

Ensemble based analyses

Light Curve Structure Importances

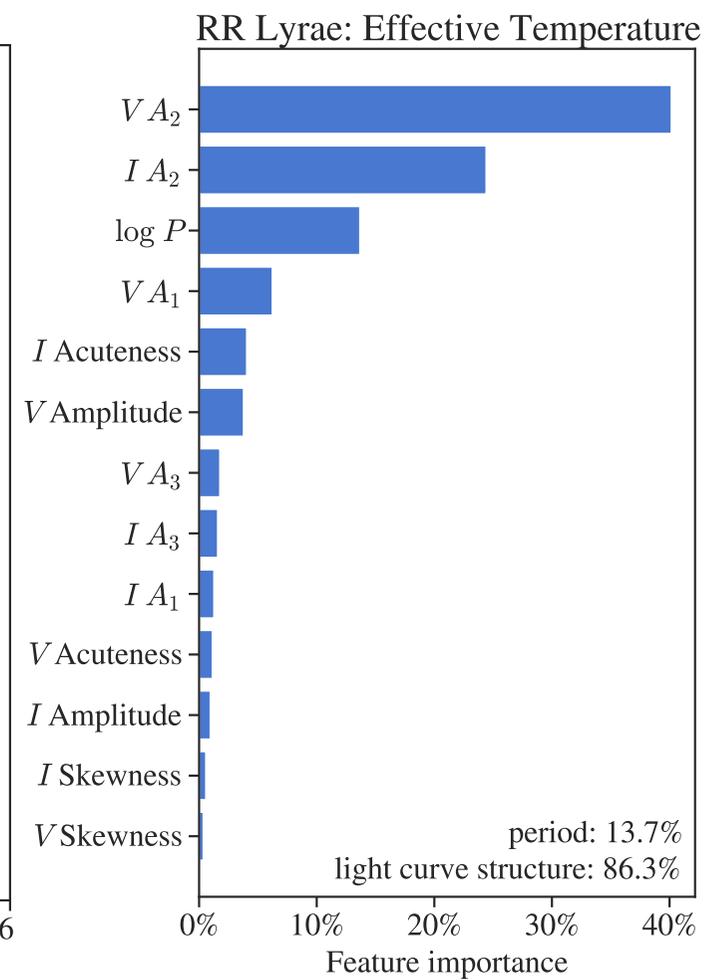
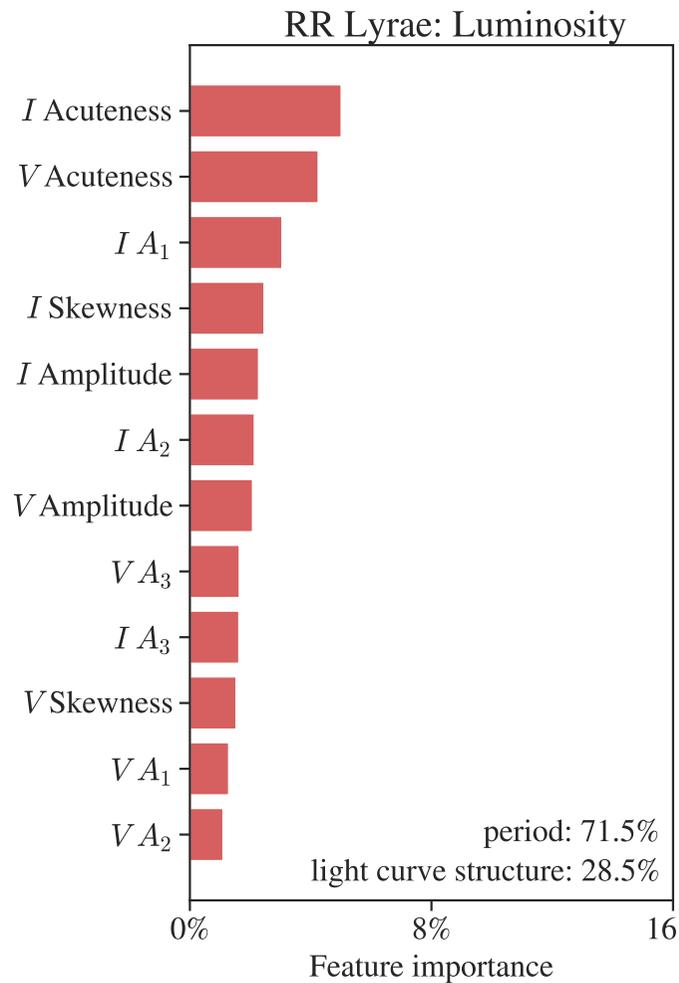
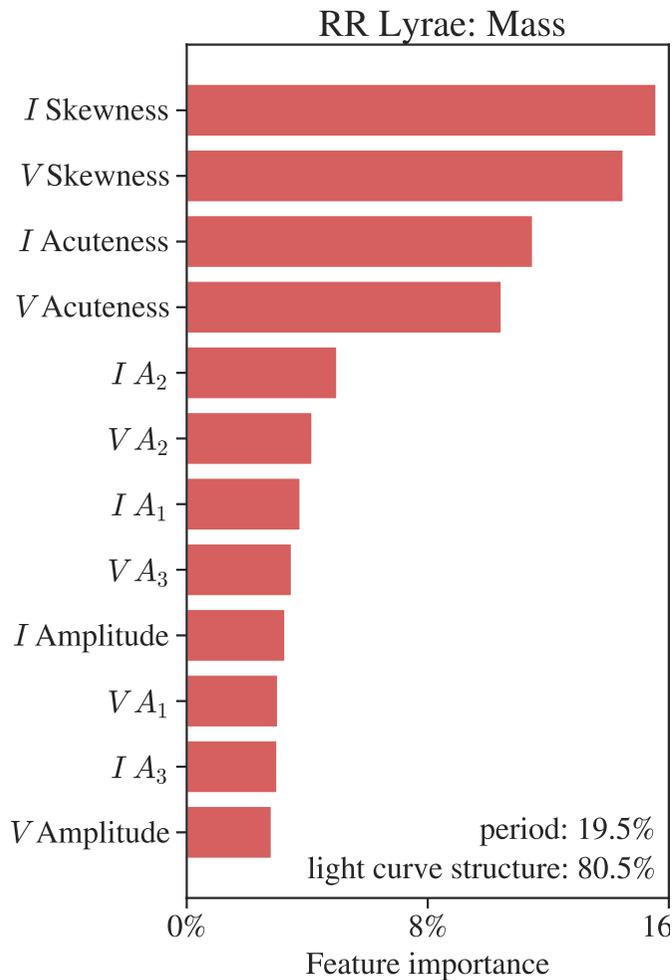


Variable	LM		ANN	
	CEP	RRL	CEP	RRL
M/M_{\odot}	0.56	0.077	0.35	0.049
$\log R/R_{\odot}$	0.017	0.027	0.011	0.019
$\log L/L_{\odot}$	0.084	0.087	0.044	0.041
T_{eff}/K	209	240	108	98
M_I	0.19	0.16	0.10	0.086
M_V	0.26	0.21	0.13	0.098
$V-I$	0.070	0.056	0.033	0.028
W	0.11	0.11	0.065	0.076

Predicted fundamental parameters

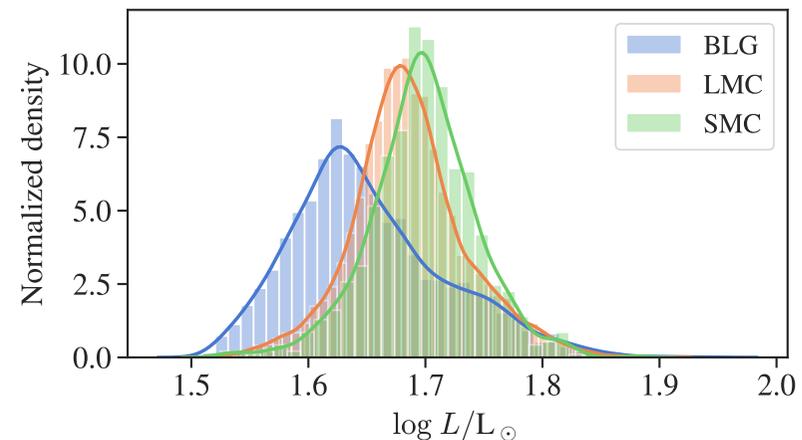
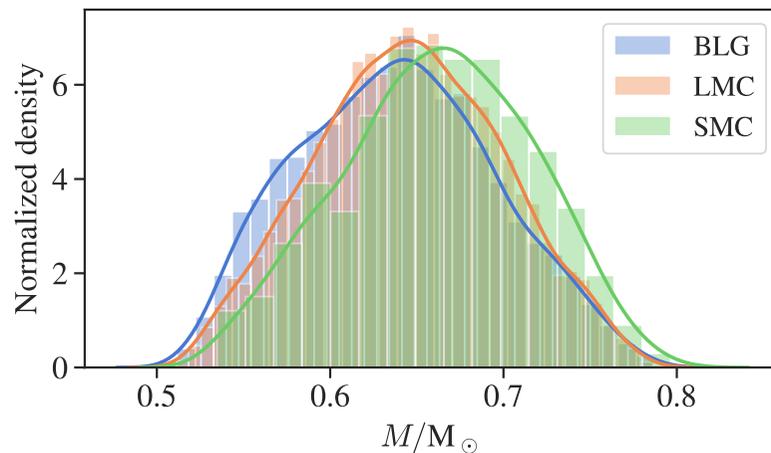
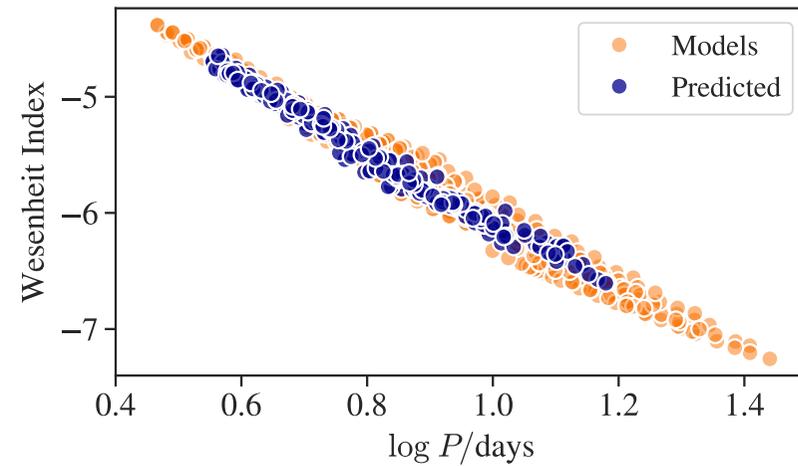
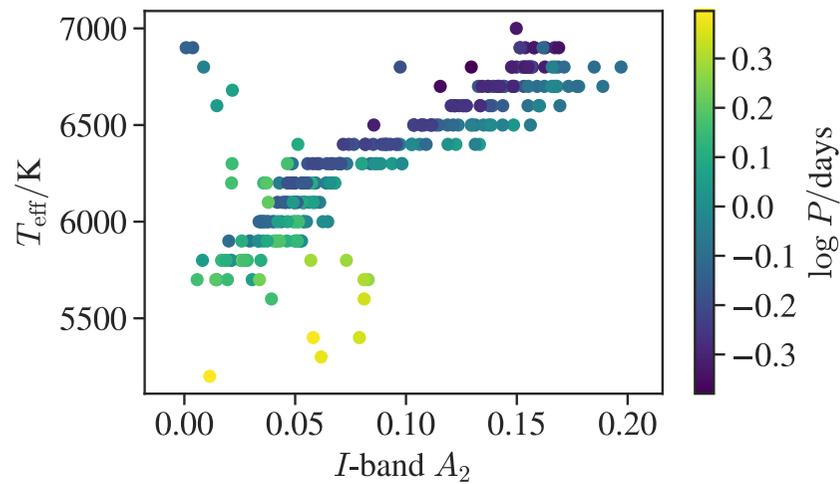
Ensemble based analyses

Which light curve structure parameters are the most important?



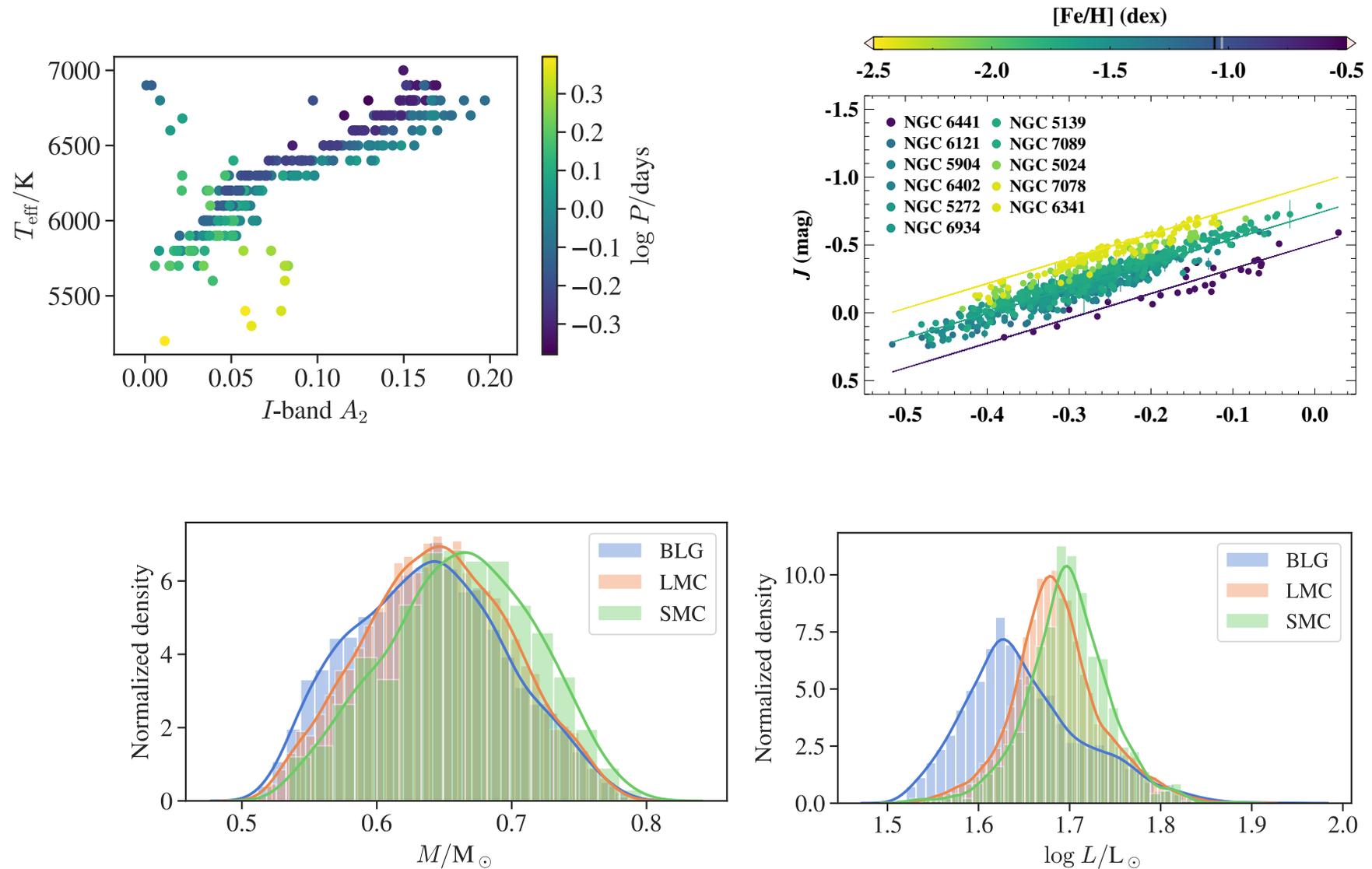
Predicted fundamental parameters

Model fitting approach to ensemble based analyses



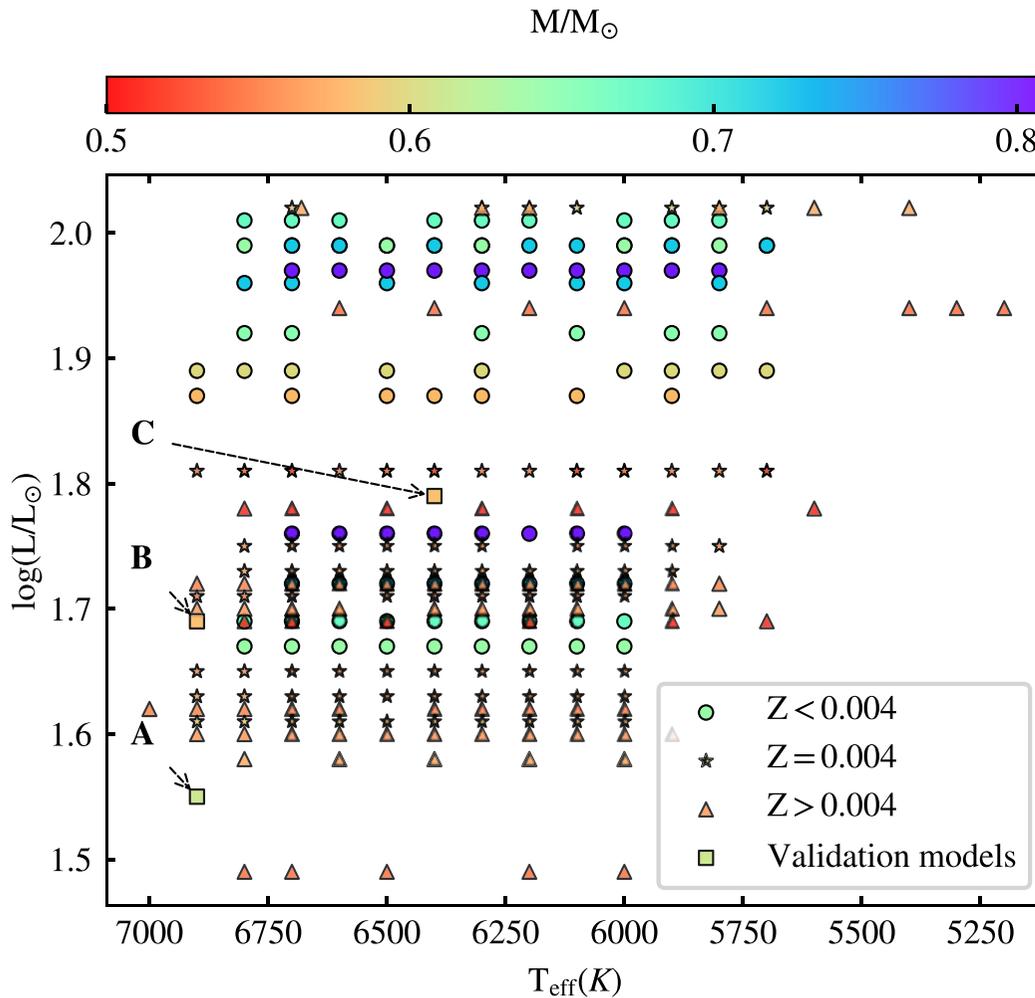
Predicted fundamental parameters

Model fitting approach to ensemble based analyses



ANN light curve interpolator

Interpolating the theoretical grid of pulsation models

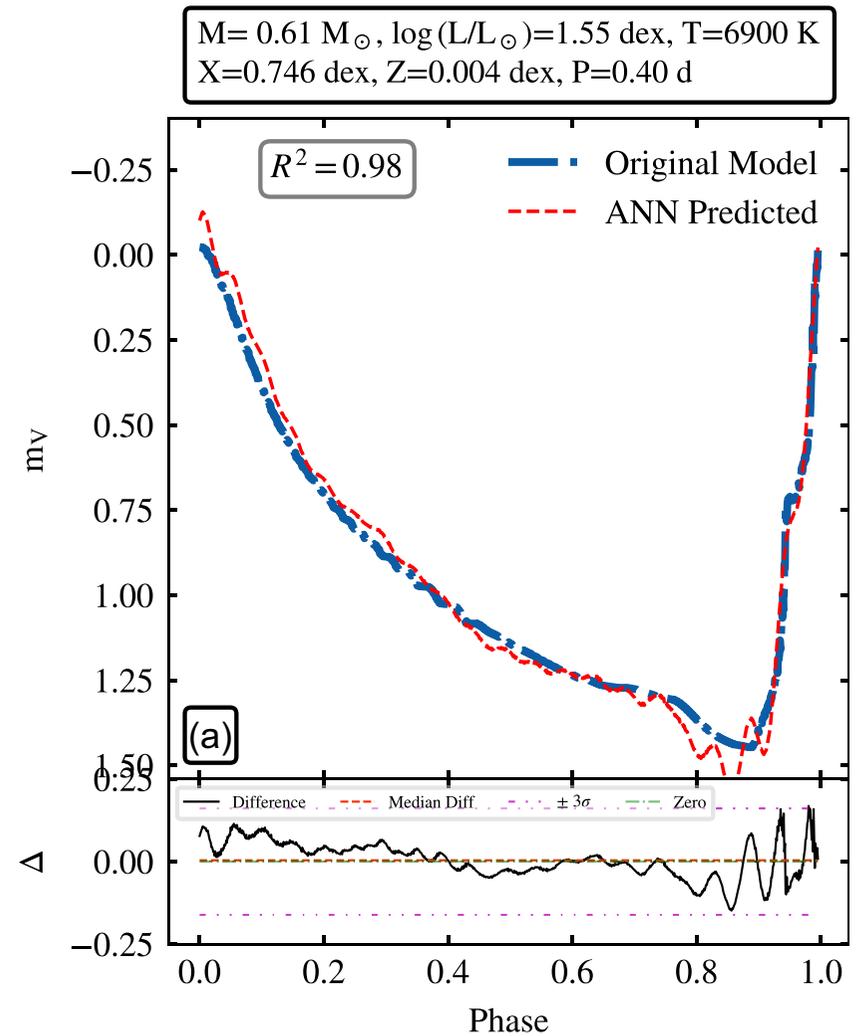
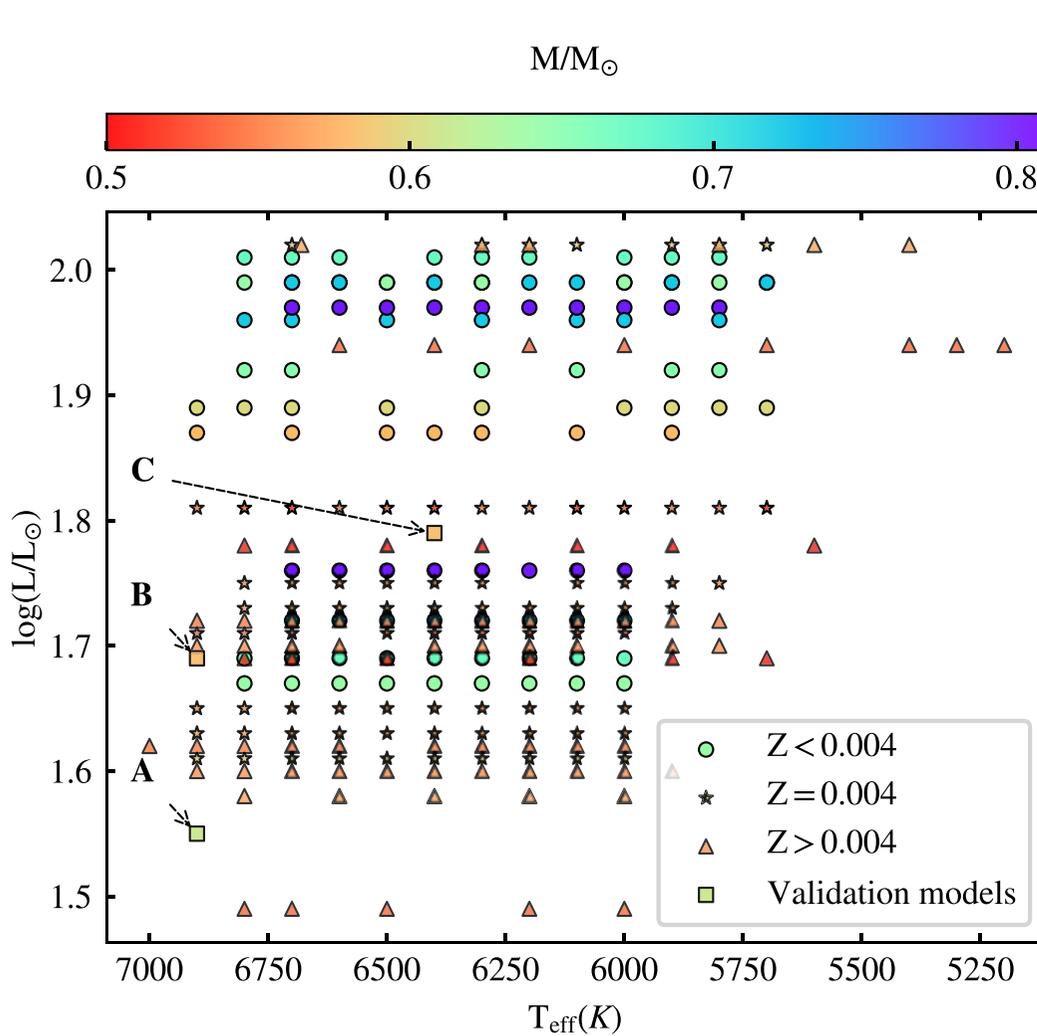


$$\mathbf{x} \equiv \left[\frac{M}{M_{\odot}}, \log\left(\frac{L}{L_{\odot}}\right), T_{\text{eff}}, X, Z, \log(P) \right],$$

S. no.	Name of hyperparameter	Possible values
1	No. of hidden layers	[1, 2, 3]
2	No of neurons in one hidden layer	[16, 32, 64, 128]
3	Optimizer	'adam' (Kingma & Ba 2014)
4	Learning rate	$[10^{-2} - 10^{-4}]$ (log sampling)
5	Activation function	['relu', 'tanh']
6	Weights initialization	'GlorotUniform'

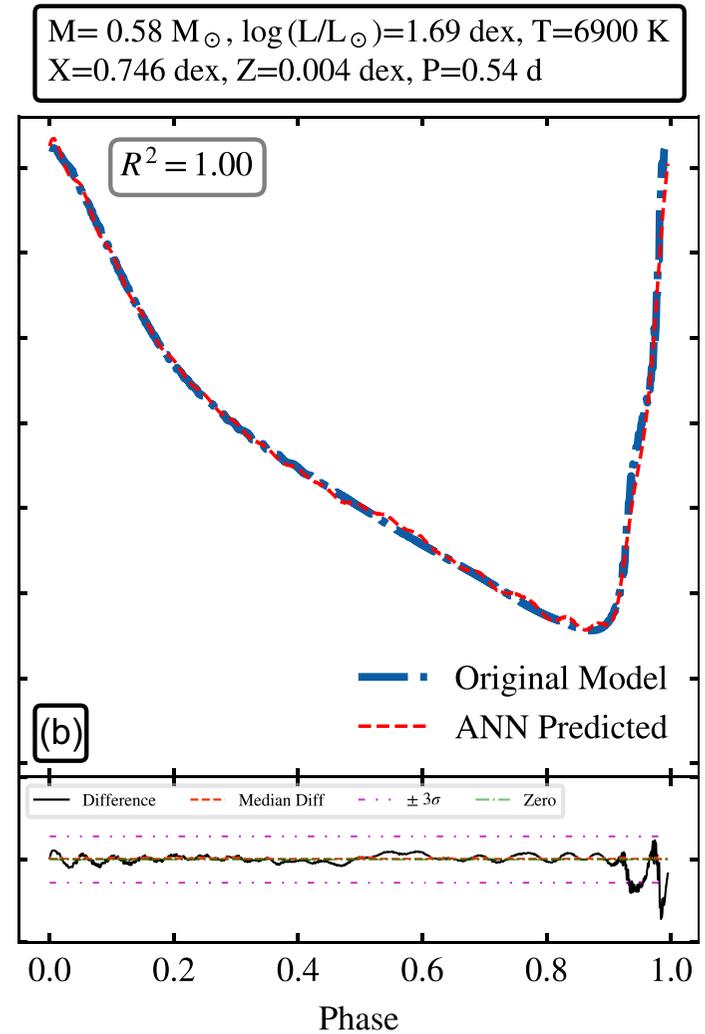
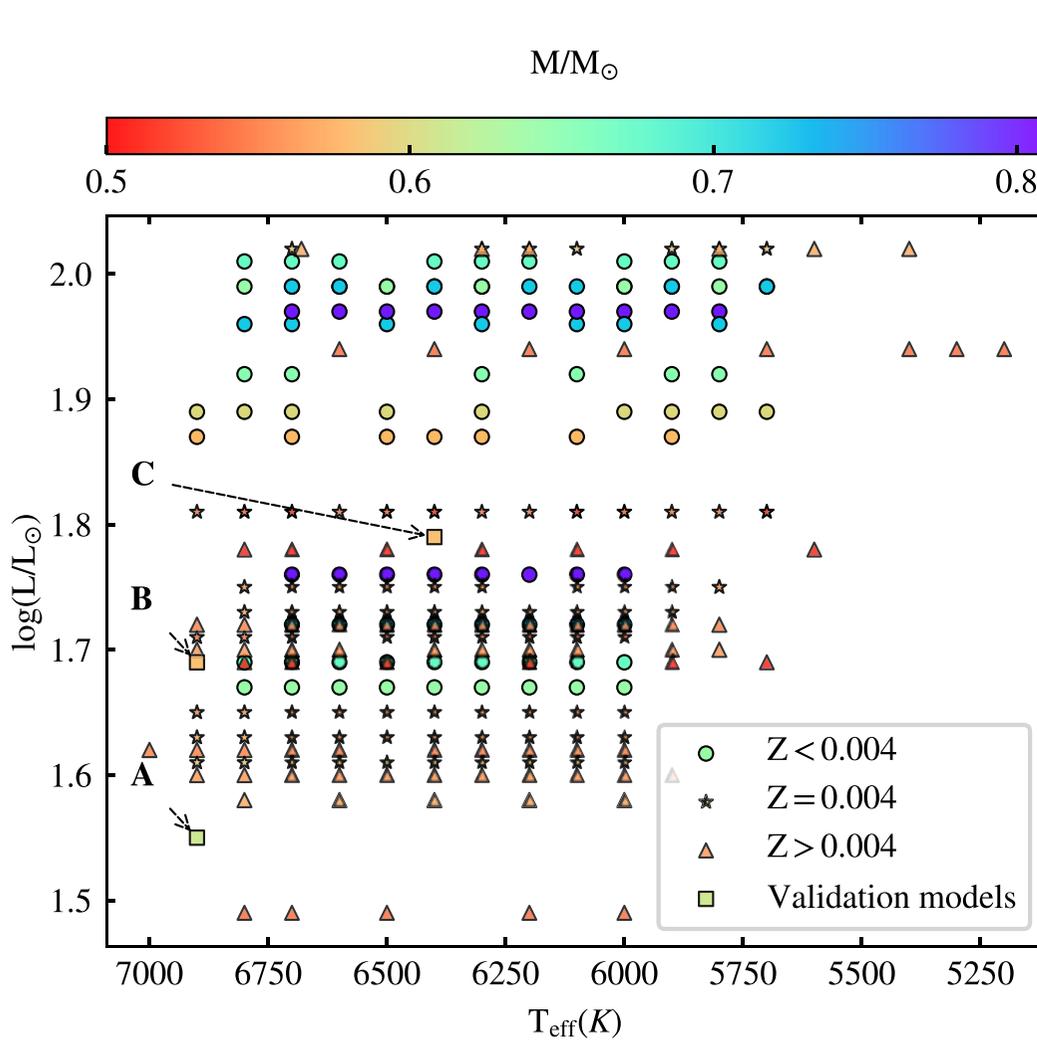
ANN light curve interpolator

Interpolating the theoretical grid of pulsation models



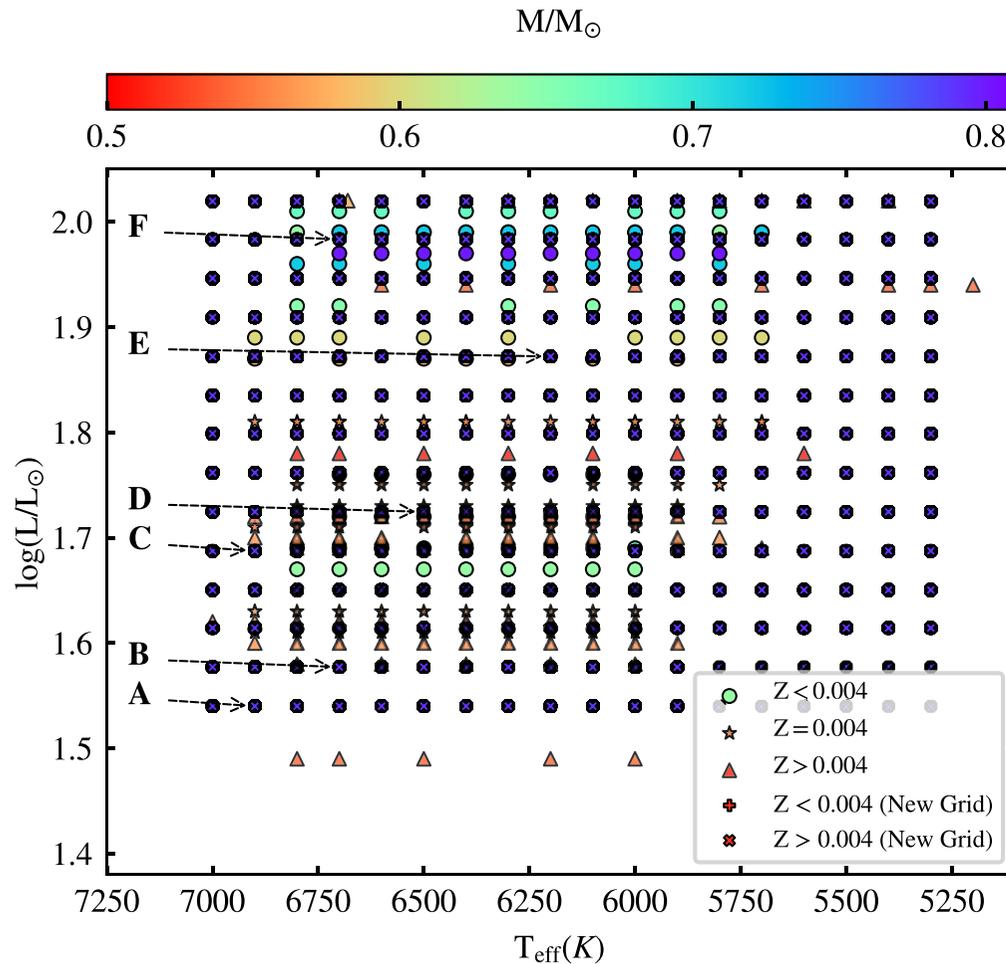
ANN light curve interpolator

Interpolating the theoretical grid of pulsation models

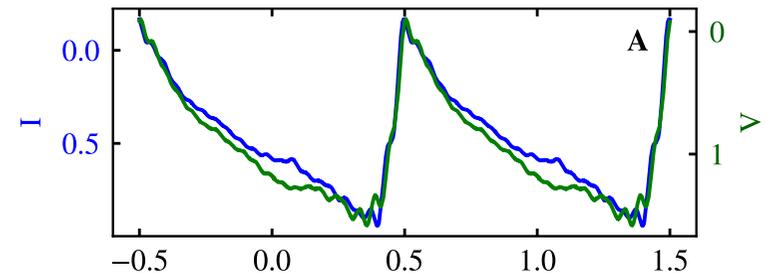


ANN light curve interpolator

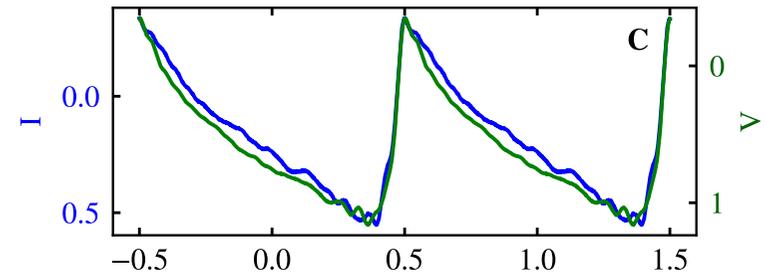
Interpolating the theoretical grid of pulsation models



$M=0.61M_{\odot}$, $\log(L/L_{\odot})=1.54$, $T_{\text{eff}} = 6900$ K
 $X = 0.7483$, $Z = 0.0067$, $P = 0.38$ d

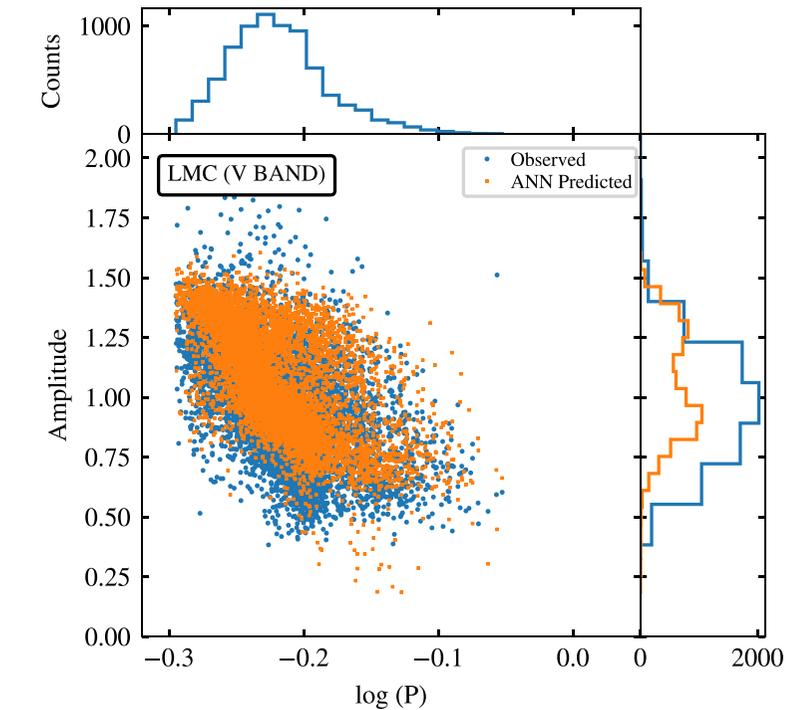


$M=0.58M_{\odot}$, $\log(L/L_{\odot})=1.69$, $T_{\text{eff}} = 6900$ K
 $X = 0.7483$, $Z = 0.0067$, $P = 0.52$ d

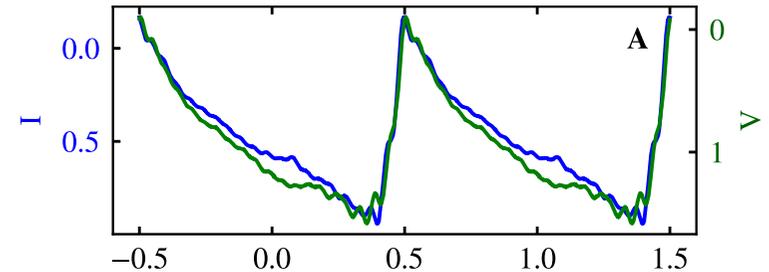


ANN light curve interpolator

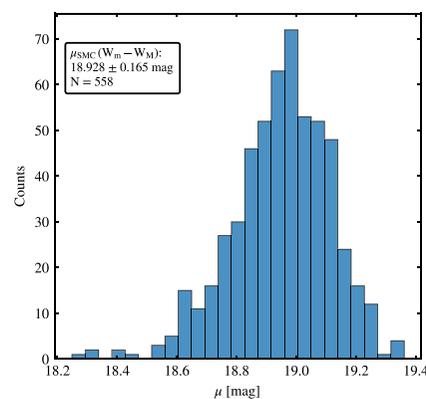
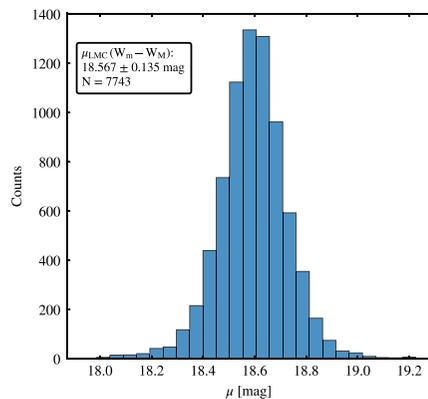
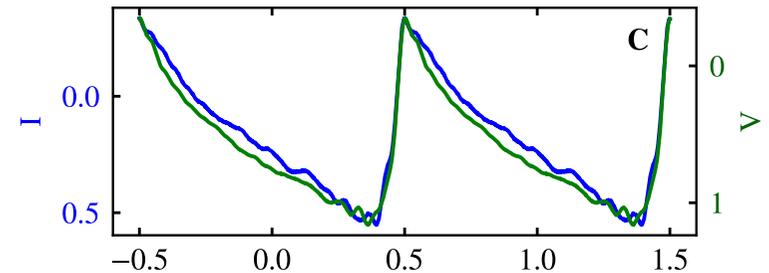
Interpolating the theoretical grid of pulsation models



$M=0.61M_{\odot}$, $\log(L/L_{\odot})=1.54$, $T_{\text{eff}} = 6900 \text{ K}$
 $X = 0.7483$, $Z = 0.0067$, $P = 0.38 \text{ d}$



$M=0.58M_{\odot}$, $\log(L/L_{\odot})=1.69$, $T_{\text{eff}} = 6900 \text{ K}$
 $X = 0.7483$, $Z = 0.0067$, $P = 0.52 \text{ d}$



ANN light curve interpolator

Generate the V band light curves of RRab stars

Please input the values of mass, luminosity, temperature, chemical composition (X,Z) and period in the specified format
It should be noted that the ANN model is an interpolator and can only predict the light curves of RRab stars with parameters within the original grid of models.
The respective range of parameters are provided against the input boxes

M :	<input type="text" value="0,61"/>	M _⊙ [0.5 - 0.8 M _⊙]
L :	<input type="text" value="1,54"/>	dex [1.49 - 2.02 dex]
T :	<input type="text" value="6900"/>	K [5200 - 7200 K]
X :	<input type="text" value="0,74832"/>	dex [0.71 - 0.7549 dex]
Z :	<input type="text" value="0,00668"/>	dex [0.0001 - 0.02 dex]
P :	<input type="text" value="0,3771"/>	day [0.1 - 2.0 day]
<input type="button" value="Predict"/>		
<input type="button" value="Reset"/>		

**** Please Reset the form before submitting the next set of parameters. ****

User may change the value of the period to get the light curve of the star with the desired period.

<https://ann-interpolator.web.app/vband.html>

V band light curve

